EE3414
Multimedia Communication Systems - I

Digital Communications Principles
Based on Lecture Notes by Elza Erkip

Yao Wang
Polytechnic University, Brooklyn, NY11201
http://eeweb.poly.edu/~yao
Outline

- Digital Communication Systems
  - Modulation of digital signals
  - Error probability vs. SNR
  - Error correction coding
  - Channel capacity: noiseless case, and noisy case
  - Advantages of digital communication
How to send digital signals?

- Digital bits -> analog waveforms (digital modulation)
  - Used in telephone modems, cell phones, digital TV, etc.
- Digital bits -> digital pulse sequences (line coding)
  - Used in computer networks
- How do we deal with channel noise?
  - Error detection (e.g. parity check)
  - Error correction coding
- How fast can we send bits?
  - Channel capacity depends on bandwidth, modulation, and SNR
    → Shannon channel capacity formula
Modulation of Digital Signals

- For transmission of digital bits over analog channels
  - Convert group of digital bits into analog waveforms (symbols)
  - The analog waveforms are formed by adapting the amplitude and/or phase of a carrier signal (ASK or PSK)
  - The carrier frequency is chosen based on the desired/acceptable operating range of the channel
  - An analog channel of bandwidth $B$ can carry at most $2B$ symbols/s. For reduced inter-symbol interference, lower than $2*B$ symbol rate is used typically
    - Shannon’s capacity formula characterizes the dependency of channel capacity on channel bandwidth and noise level
  - Equalizer is used at the receiver to reduce the inter-symbol interference
A Simple Example

- Digital information: Sequence of 0’s and 1’s: 001101…..
- One bit every T seconds. During $0 < t < T$
  - To send a 0, send $s_0(t) = A \cos(2\pi f_c t)$
  - To send a 1, send $s_1(t) = -A \cos(2\pi f_c t)$

- Input signal

- Modulated signal

- This is called Binary Pulse Amplitude Modulation (PAM) or Binary Phase Shift Keying (BPSK).
- For a channel with bandwidth $B$, $T \geq \frac{1}{2B}$, to avoid inter-symbol interference
Amplitude Shift Keying (ASK)

M-ary ASK: each group of \( \log_2 M \) bits generates a symbol. The number corresponding to the symbol controls the amplitude of a sinusoid waveform. The number of cycles in the sinusoid waveform depends on the carrier frequency. (Also known as Pulse Amplitude Modulation or PAM)

4-ASK: 2 bits/symbol (00=-3, 01=-1, 11=1, 10=3)

Example: Given a sequence: 01001011…, what is the analog form resulting from 4-ASK?

Symbol representation: “-1”, ”-3”, ”3”, ”1”

Waveform:
8-ASK: 3 bits/symbol (000=-7, 001=-5, 011=-3, 010=-1, 110=1, 111=3, 101=5, 100=7)

The mapping from bits to symbols are done so that adjacent symbols only vary by 1 bit, to minimize the impact of transmission error (this is called Gray Coding)
Quadrature Amplitude Modulation (QAM)

M-ary QAM uses symbols corresponding to sinusoids with different amplitude as well as phase, arranged in the two-dimensional plane.

Ex. 4-QAM (only phase change):

\[\begin{align*}
00 &= \cos(\omega_c t - \pi/4) \\
01 &= \cos(\omega_c t - 3\pi/4) \\
10 &= \cos(\omega_c t - 7\pi/4) \\
11 &= \cos(\omega_c t - 5\pi/4)
\end{align*}\]

Note this is equivalent to analog QAM if we interpret the first bit and second bit coming from two pulse sequences!
Example of 4-QAM

**Example:** Given a sequence: 01001011…, what is the analog form resulting from 4-ASK?

Using the previous mapping, the analog waveform for the above sequence is

![Waveform Diagram](image)
16-QAM, etc.

16 QAM (4 bits/symbol):

64-QAM (6 bits/symbol)
Vector Representation

- For BPAM, In $s_0(t)$ and $s_1(t)$ the term $\cos(2\pi f_c t)$ is common
  - We can represent $s_0(t) = A$, $0 < t < T$
  
  
  $s_1(t) = -A$, $0 < t < T$

- Or

Ex: A digital sequence 100110 is sent as \{A –A –A A A –A –A\}
Vector Representation for ASK

- M-ary ASK (M=4, send two bits at a time)

  - Labeling is done in such a way that adjacent points only differ in one bit (called Gray mapping)

Ex: A digital sequence 100010 is sent as \{-3A, 5A,\ldots\}
Two Dimensional Modulation

- Modulated signal: \( s_i(t) = A_c \cos(2\pi f_c t) + A_s \sin(2\pi f_c t) \)
  - 4-QAM (Quadrature Amplitude Modulation)
    - 8-PSK (Phase Shift Keying)
Parameters of Modulation

- Three important parameters of a modulation scheme:
  - Minimum distance $d_{\text{min}}$: The smallest distance among points in vector representation, which affects error detection capability
  - Average energy $E_{\text{av}}$
  - Number of bits/symbol = $\log_2(M)$

- Example:
  - 4-ASK: $d_{\text{min}}=2A$, $E_{\text{av}}=2(A^2+9A^2)/4=5A^2$, $\log_2(M)=2$
  - 4-QAM: $d_{\text{min}}=2A$, $E_{\text{av}}=2A^2$, $\log_2(M)=2$
  - For the same $E_{\text{av}}$ and $M$, we want to maximize $d_{\text{min}}$ (minimize effect of transmission noise)
  - For the same $d_{\text{min}}$ and $M$, we want to minimize $E_{\text{av}}$ (minimize power consumption)
Other Digital Modulation Techniques

Original signal (in a pulse sequence)

Amplitude shift-keying (ASK)

Frequency shift-keying (FSK)

Phase shift-keying (PSK)
Effect of noise

- Simple channel: Additive White Gaussian

\[
p(x / m \text{ is sent}) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}
\]

• Noise \( n(t) \) is Gaussian
Optimum Receiver for AWGN

- Received signal amplitude $r$
  
  $r = s + n$, $s = A$ or $-A$,
  Left is the pdf of $r$, assuming $s$ is equally likely to be $A$ and $-A$

- Decide $A$ ("1") was sent if $r > 0$, $-A$ ("0") was sent if $r < 0$.

- How often do we make errors?
  - Depends on the distance $d_{\text{min}} = 2A$
    - If $s = A$, $n$ must be $< -d_{\text{min}}/2$
    - If $s = -A$, $n$ must be $> d_{\text{min}}/2$

  - The larger is $d_{\text{min}}$, the less likely
Error Probability of BPSK

- To have few errors we want $A$ large compared to $\sigma$

- Signal to noise ratio $= \text{SNR} = \frac{A^2}{\sigma^2}$

- Probability of error

$$P_e = P(A) \int_{-\infty}^{-A} \frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2} \, dx + P(-A) \int_{A}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2} \, dx$$

If $P(A) = P(-A) = 1/2$

$$P_e = \int_{A}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2} \, dx = \int_{A/\sigma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx = Q(\sqrt{\text{SNR}})$$

$P_e = Q(\sqrt{\text{SNR}})$
Channel Error Detection

- By adding additional bits to the information bits, we can also detect errors
- Parity Check (check-sum)
  - Append a parity bit to the end of each block (called a frame) of k information bits such that the total number of '1' is even or odd
    - Used in IP packet for error detection because of simplicity
- Example:
  - ASCII "G" = 1 1 1 0 0 0 1
  - with even parity = append ?
  - with odd parity = append ?
Error Detection Probability

- Parity check can detect all single bit errors in a block (if fact all error patterns with odd number of error bits)
- Cannot detect double errors or any even number of errors.
- Probability of k error bits in a n-bit frame (assuming bit error rate \( p_b \))

\[
p_n(k) = \binom{n}{k} p_b^k (1 - p_b)^{n-k}
\]

Ex: if \( n = 10^4 \), \( p_b = 10^{-6} \) (a good channel, e.g. ISDN)

\[
p_n(1) \approx 10^{-2}, p_n(2) \approx \frac{1}{2} 10^{-4} \leq p_n(1)
\]

if \( n = 10^3 \), \( p_b = 10^{-3} \) (a not so good channel, e.g. wireless)

\[
p_n(1) \approx 1, p_n(2) \approx \frac{1}{2}
\]
Cyclic-Redundancy Codes (CRC)

General Method:

- The transmitter generates an *t*-bit check sequence number from a given *k*-bit data frame such that the resulting *(k+t)*-bit frame is divisible by some number.
- The receiver divides the incoming frame by the same number.
- If the result of the division does not leave a remainder, the receiver assumes that there was no error.
- If *n* is large, undetectable error patterns are very unlikely.
- Widely used in data communications.
Error Correction by Channel Coding

• In addition to detect errors, we can correct errors by inserting parity bits. This is called channel coding.

• Simplest channel code: Repetition coding:
  – Instead of sending one 0, send three 0’s.
  – If we receive more 0’s than 1’s: Decide 000 was sent
  – Error probability=3 or 2 bits are wrong=
    • =probability of receiving 111,110,101,011
    • Smaller than Pe
    • Example: Old Pe=10^-2, New Pe=3*10^-4
  – But the information rate is decreased
    • Instead of R=1bit/sec, we have R=1/3 bits/sec

• More sophisticated channel codes can correct the same number of errors with lower redundancy (less reduction in information rate)
  – Block codes (Reed-Solomn codes), convolutional codes, turbo codes

\[ P_e^3 + 3(1 - P_e)P_e^2 \approx 3P_e^2 \]
Channel Capacity

- Channel capacity = maximum number of bits/second we can send for reliable transmission
- What determines this?
  - The receiver typically evaluate the received signal level over entire bit interval, to determine which bit is sent (0 or 1)
  - If we sent more bits/second, the interval of each bit is shorter, channel noise will more easily make the receiver to make mistakes
  - With higher order modulation, we decode multiple bits together over each symbol interval

```
  0  0  1  1  0  1
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![Waveform](image)
Noiseless Case

- To send a digital sequence, we send a sequence of pulses
  - level A representing “1”, and level –A for “0”
- If the interval of each pulse is $T$, what is the maximum frequency?
  - The maximum frequency occurs when we have alternating 1 and −1, spanning $2T$ time per cycle
  - This signal has a maximum frequency of $f_{\text{max}} = 1/2T$
  - With a channel with bandwidth $B$, $f_{\text{max}} = 1/2T \leq B$
    - $T \geq 1/2B$, $C = 1/T \leq 2B$
    - we can send at most $C = 2B$ bits/second!
- Instead of two-level pulse, we can use $N$-level pulse ($m = \log_2 N$ bits/pulse), we can send $C = 2Bm$ bits/second (Nyquist channel capacity)
- By increasing $N$ (hence $m$), we can reach infinite bit rate!
  - What is wrong?
  - Here we assume the channel is noiseless, every level can be distinguished correctly
Channel Capacity (Noisy Case)

- When there are more levels in a pulse, the signal difference between two adjacent levels is smaller (for the same total dynamic range). Noise in communication channel is more likely to make the received/detected level differ from the actual level.

- The channel capacity depends on the signal to noise ratio (SNR)
  - SNR = signal energy / noise energy

- Channel Channel Capacity
  - \( C = B \log_2 (1 + SNR) \) bits/second

- Ex: \( B = 6 \text{MHz}, \ SNR = 20 \text{ dB} = 100 \), \( C = ? \)
Advantages of Digital Communication

- More tolerant to channel noise.
  - With amplitude shift keying: as long as the noise does not change the amplitude from one level to another level, the original bits can be inferred
  - With QAM: as long as the received signal is more close to the original symbol than its neighboring symbols
  - Each repeater can regenerate the analog modulated signals from demodulated bits (noise do not accumulate)
- Can insert parity bits before sending data to allow detection/correction of errors
- Can apply digital compression techniques to reduce data rate subject to distortion criterion
- Different signals can be multiplexed more easily
  - Internet packets can contain any types of signals, cell phones can send different types of data
What Should You Know

- What is digital modulation?
- One and two dimensional modulation schemes
- Parameters of digital modulation schemes
  - Can calculate basic parameters for a given modulation scheme
  - Understand the design objective
- Channel error detection and correction
  - Should understand how simple parity check works and how repetition coding works
- Channel capacity
  - Understand the role of channel bandwidth and SNR in determining channel capacity
- Advantages of digital communications
References