
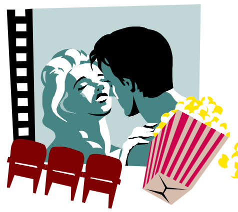
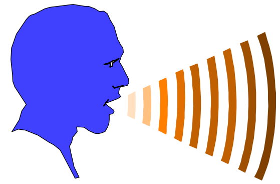
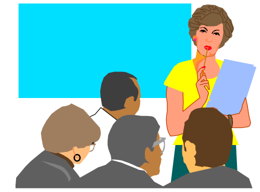

EE3414
Multimedia Communication Systems - I

Digital Communications Principles

Based on Lecture Notes by Elza Erkip

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Outline

- Digital Communication Systems
  - Modulation of digital signals
  - Error probability vs. SNR
  - Error correction coding
  - Channel capacity: noiseless case, and noisy case
  - Advantages of digital communication

How to send digital signals?

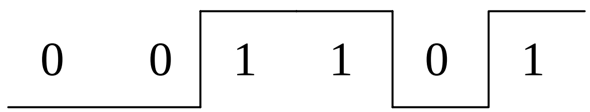
- Digital bits -> analog waveforms (digital modulation)
  - Used in telephone modems, cell phones, digital TV, etc.
- Digital bits -> digital pulse sequences (line coding)
  - Used in computer networks
- How do we deal with channel noise?
  - Error detection (e.g. parity check)
  - Error correction coding
- How fast can we send bits ?
  - Channel capacity depends on bandwidth, modulation, and SNR
    → Shannon channel capacity formula

Modulation of Digital Signals

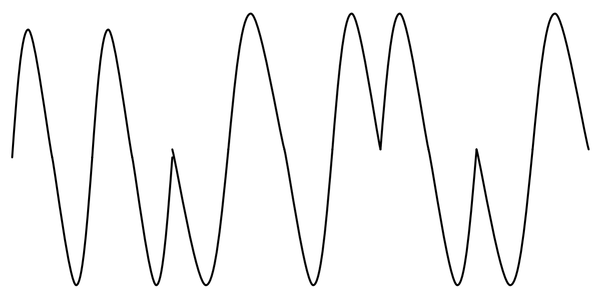
- For transmission of digital bits over analog channels
  - Convert group of digital bits into analog waveforms (symbols)
  - The analog waveforms are formed by adapting the amplitude and/or phase of a carrier signal (ASK or PSK)
  - The carrier frequency is chosen based on the desired/acceptable operating range of the channel
  - An analog channel of bandwidth B can carry at most 2B symbols/s. For reduced inter-symbol interference, lower than 2*B symbol rate is used typically
    - Shannon's capacity formula characterizes the dependency of channel capacity on channel bandwidth and noise level
  - Equalizer is used at the receiver to reduce the inter-symbol interference

A Simple Example

- Digital information: Sequence of 0's and 1's: 001101…..
- One bit every T seconds. During 0 < t < T
  - To send a 0, send $s_0(t) = A\cos(2\pi f_c t)$
  - To send a 1, send $s_1(t) = -A\cos(2\pi f_c t)$
- Input signal



- Modulated signal



- This is called Binary Pulse Amplitude Modulation (PAM) or Binary Phase Shift Keying (BPSK).
- For a channel with bandwidth B, T >=1/2B, to avoid inter-symbol interference

Amplitude Shift Keying (ASK)

M-ary ASK: each group of $\log_2 M$ bits generates a symbol. The number corresponding to the symbol controls the amplitude of a sinusoid waveform. The number of cycles in the sinusoid waveform depends on the carrier frequency.
(Also known as Pulse Amplitude Modulation or PAM)

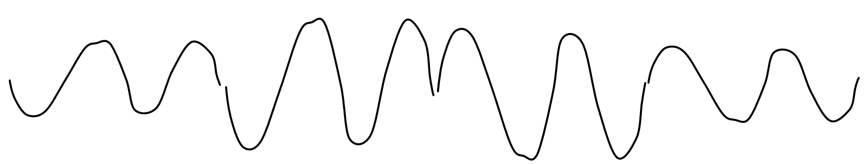
4-ASK: 2 bits/symbol (00=-3, 01=-1, 11=1, 10=3)

"00"(-3A) "01"(-A) "11" (A) "10"(3A)

Example: Given a sequence: 01001011…, what is the analog form resulting from 4-ASK?

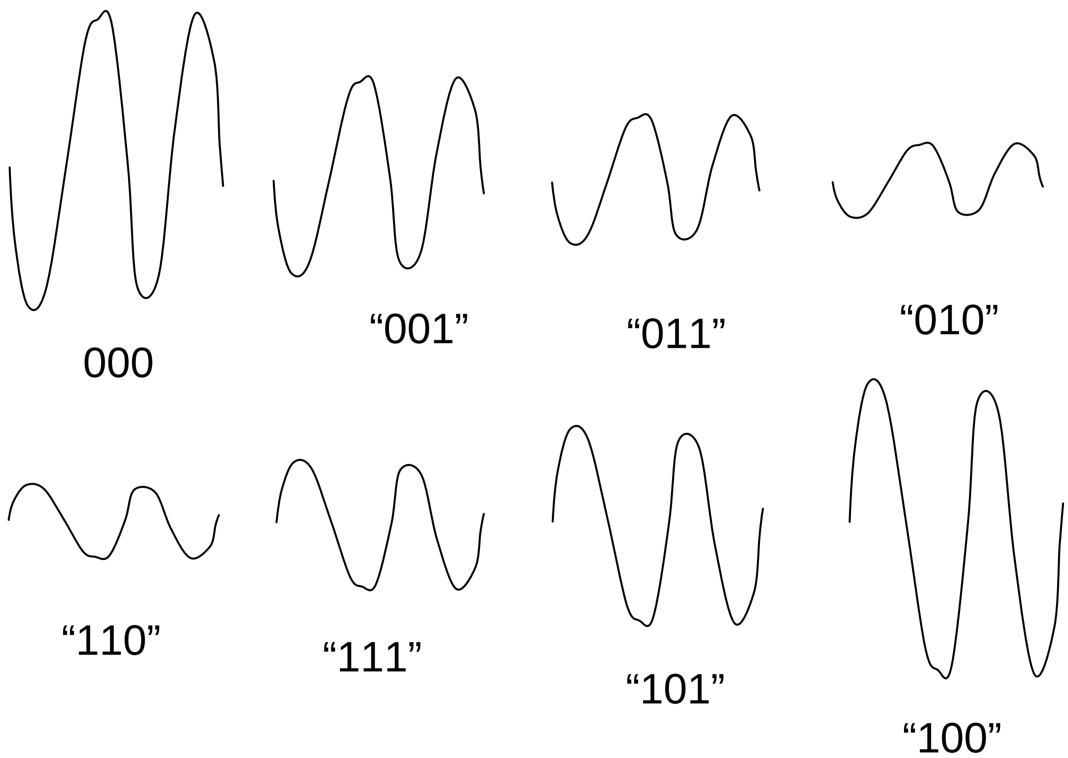
Symbol representation: "-1","-3","3","1"

Waveform:



8-ASK

8-ASK: 3 bits/symbol (000=-7, 001=-5, 011=--3, 010=-1, 110=1, 111=3, 101=5, 100=7)

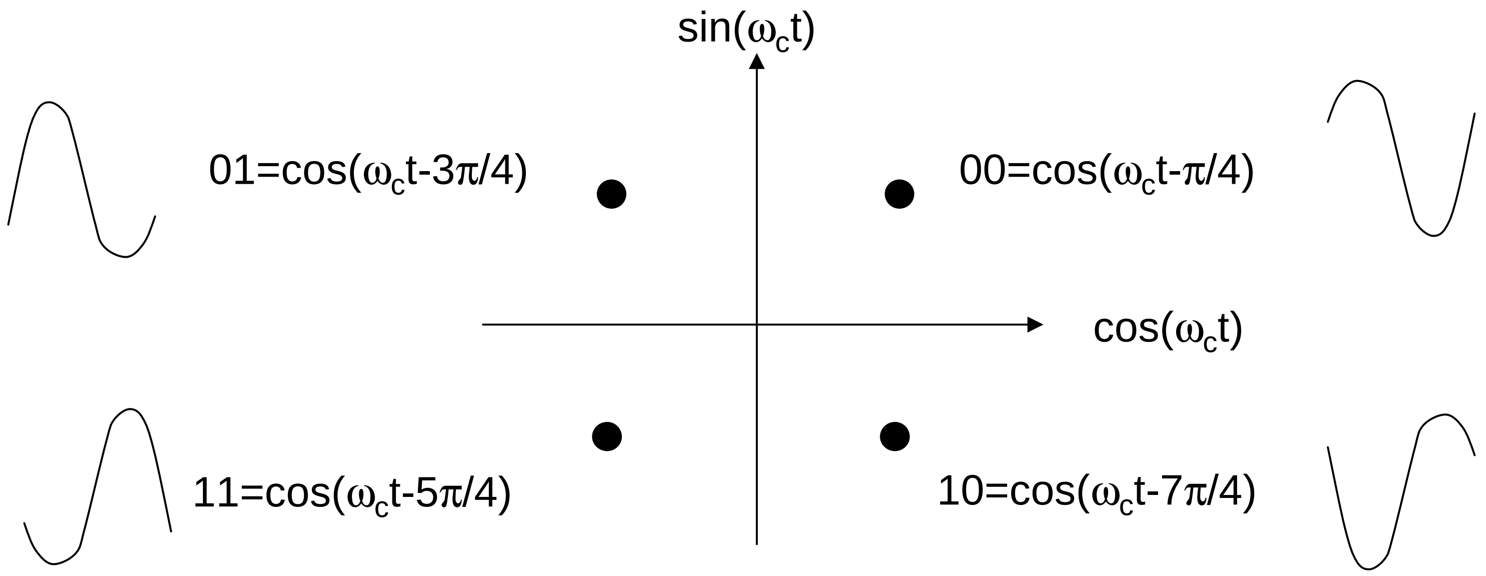


The mapping from bits to symbols are done so that adjacent symbols only vary by 1 bit, to minimize the impact of transmission error (this is called Gray Coding)

Quadrature Amplitude Modulation (QAM)

M-ary QAM uses symbols corresponding to sinusoids with different amplitude as well as phase, arranged in the two-dimensional plane.

Ex. 4-QAM (only phase change):

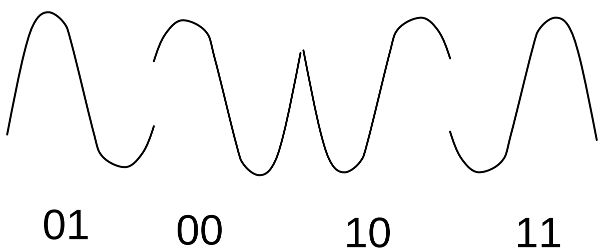


Note this is equivalent to analog QAM if we interpret the first bit and second bit coming from two pulse sequences!

Example of 4-QAM

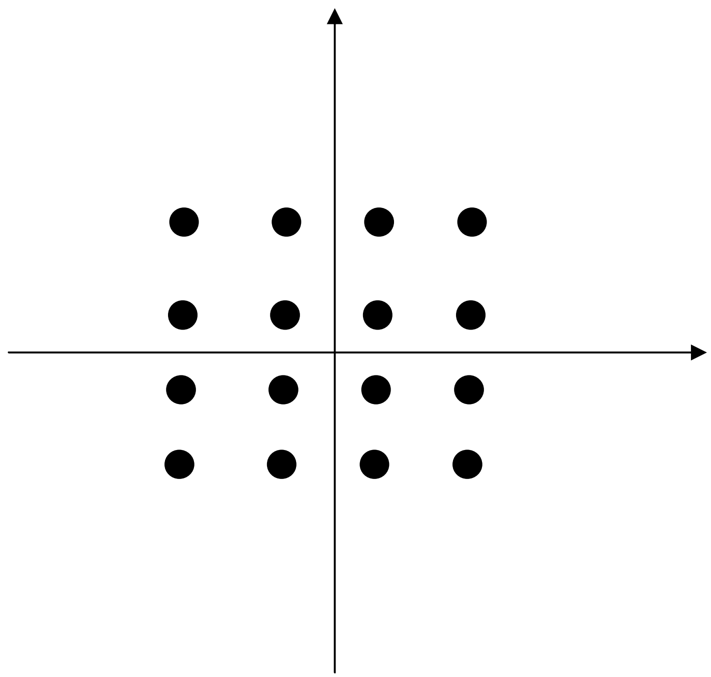
Example: Given a sequence: 01001011…, what is the analog form resulting from 4-ASK?

Using the previous mapping, the analog waveform for the above sequence is

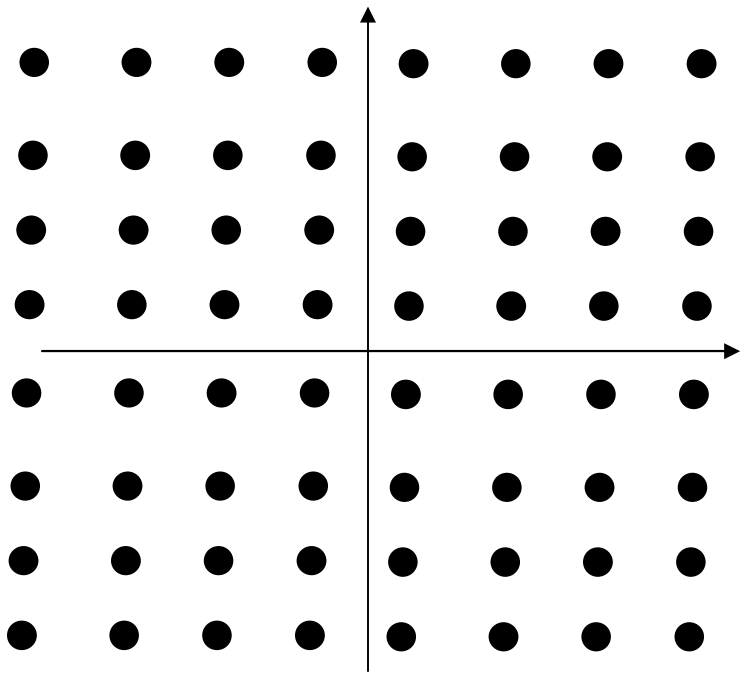


16-QAM, etc.

16 QAM (4 bits/symbol):

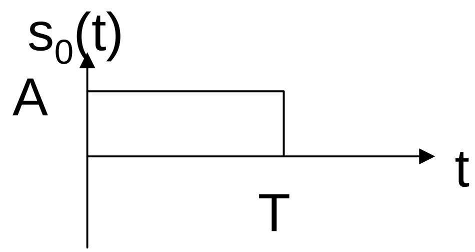


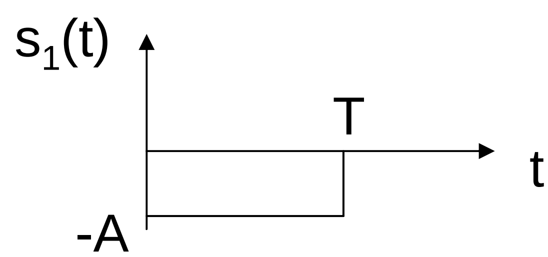
64-QAM (6 bits/symbol)



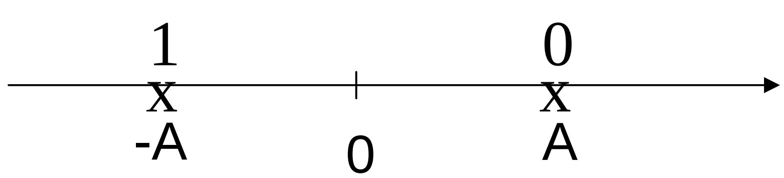
Vector Representation

- For BPAM, In $s_0(t)$ and $s_1(t)$ the term $\cos(2\pi f_c t)$ is common
  - We can represent $s_0(t) = A, 0<t<T$
    $s_1(t) = -A, 0<t<T$





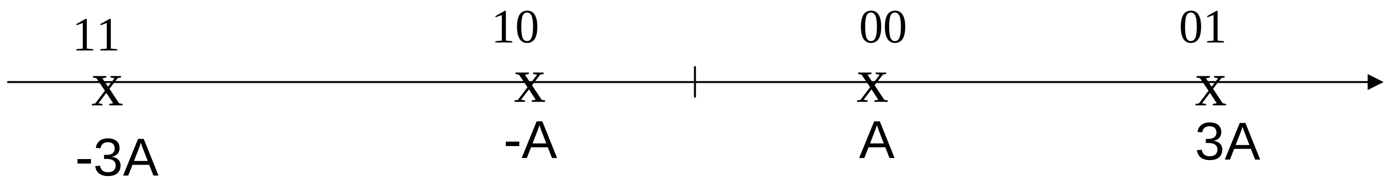
  - Or



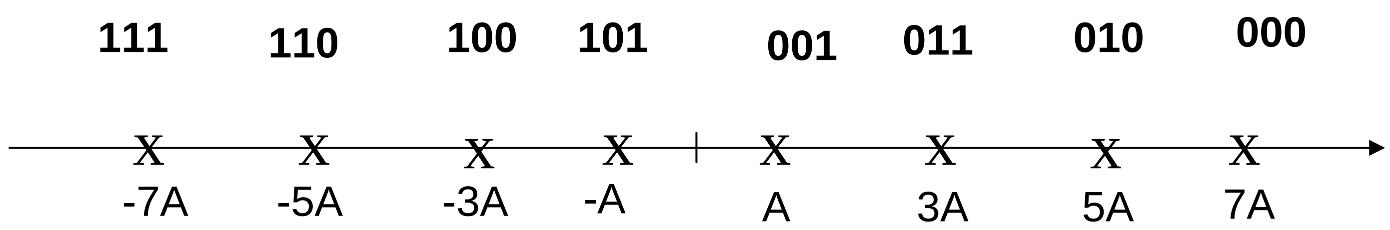
Ex: A digital sequence 100110 is sent as {A –A –A A A –A –A}

Vector Representation for ASK

- M-ary ASK (M=4, send two bits at a time)



- Labeling is done in such a way that adjacent points only differ in one bit (called Gray mapping)

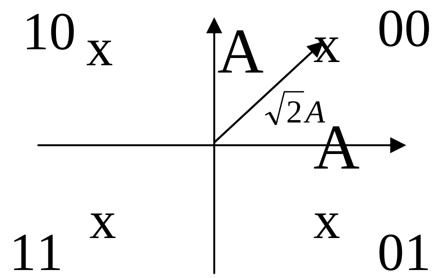


8-ASK

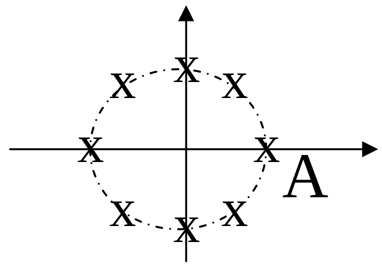
Ex: A digital sequence 100010 is sent as {-3A , 5A,…}

Two Dimensional Modulation

- Modulated signal $s_i(t) = A_c \cos(2\pi f_c t) + A_s \sin(2\pi f_c t)$
  - 4-QAM (Quadrature Amplitude Modulation)



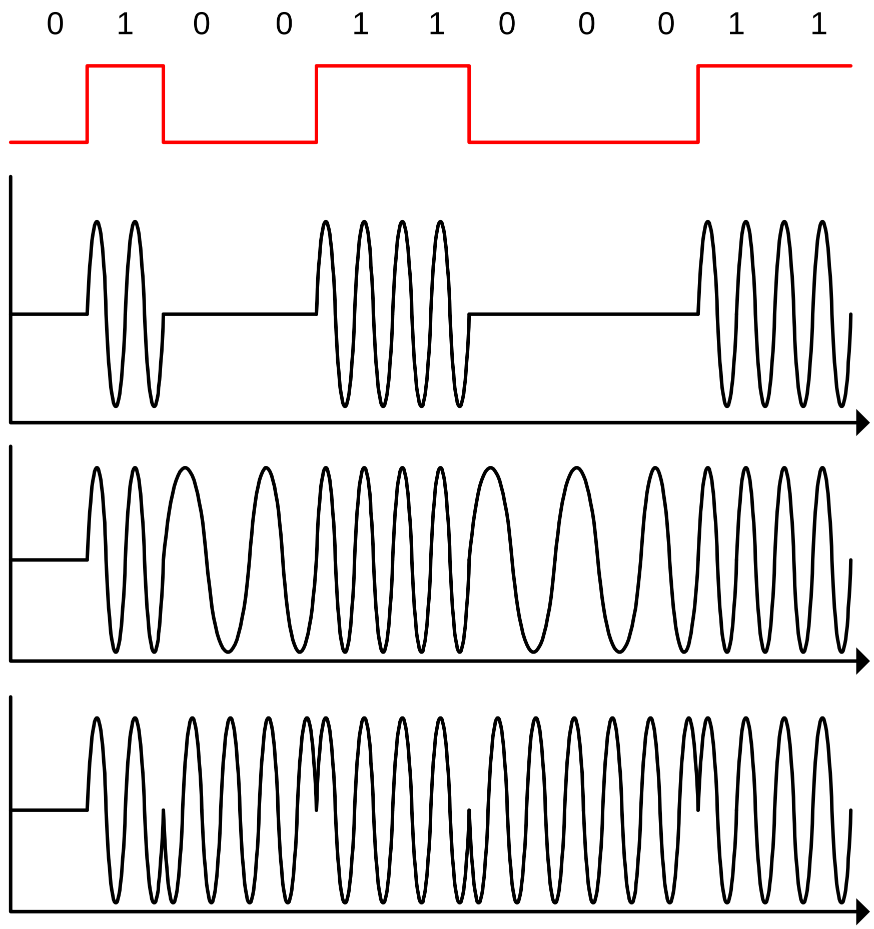
  - 8-PSK (Phase Shift Keying)



Parameters of Modulation

- Three important parameters of a modulation scheme:
  - Minimum distance d_{min}: The smallest distance among points in vector representation, which affects error detection capability
  - Average energy E_{av}
  - Number of bits/symbol= $\log_2(M)$
- Example:
  - 4-ASK: d_{min}=2A, $E_{av}=2(A^2+9A^2)/4=5A^2$, $\log_2(M)=2$
  - 4-QAM: d_{min}=2A, $E_{av}=2A^2$, $\log_2(M)=2$
  - For the same E_{av} and M, we want to maximize d_{min} (minimize effect of transmission noise)
  - For the same d_{min} and M, we want to minimize E_{av} (minimize power consumption)

Other Digital Modulation Techniques



Original signal (in a pulse sequence)

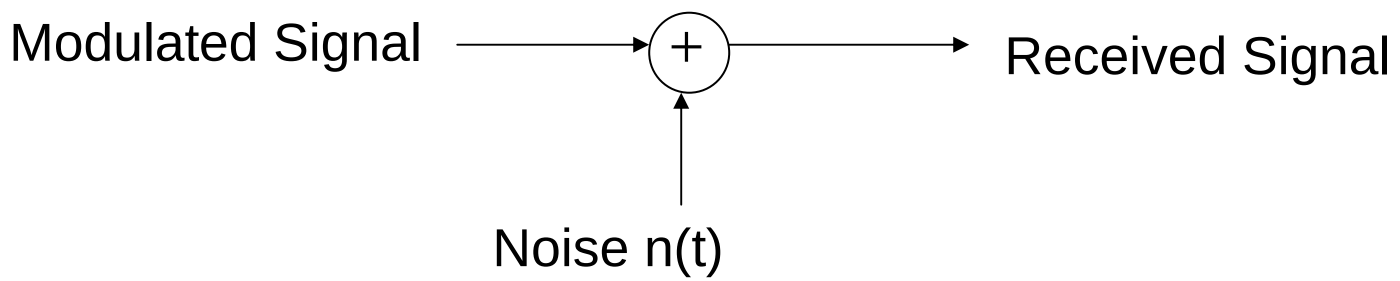
Amplitude shift-keying (ASK)

Frequency shift-keying (FSK)

Phase shift-keying (PSK)

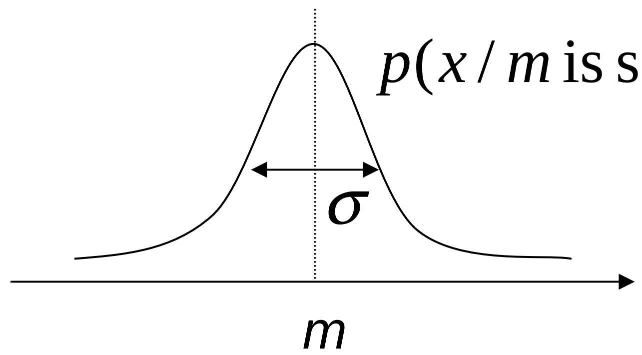
Effect of noise

- Simple channel: Additive White Gaussian



- Noise n(t) is Gaussian

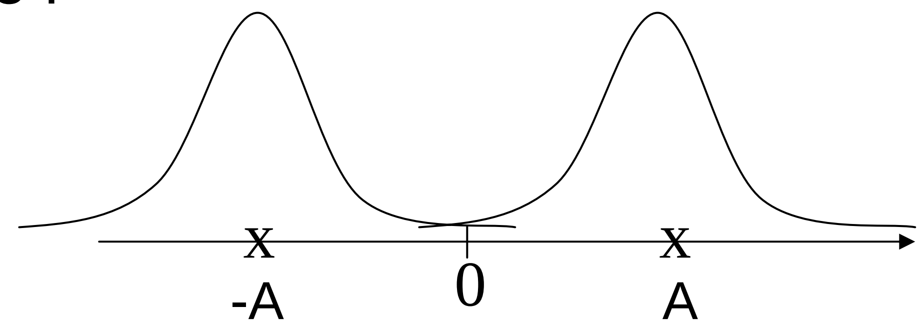
$$p(x / m \text{ is sent}) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-m)^2/2\sigma^2}$$



Optimum Receiver for AWGN

- Received signal amplitude r

r=s+n, s=A or –A,
Left is the pdf of r, assuming s is equally likely to be A and -A



- Decide A ("1") was sent if r>0, -A ("0") was sent if r<0.
- How often do we make errors?
  - Depends on the distance d_{min}=2A
    - If s= A, n must be < $-d_{min}/2$
    - If s=-A, n must be > $d_{min}/2$
  - The larger is $d_{min,}$ the less likely

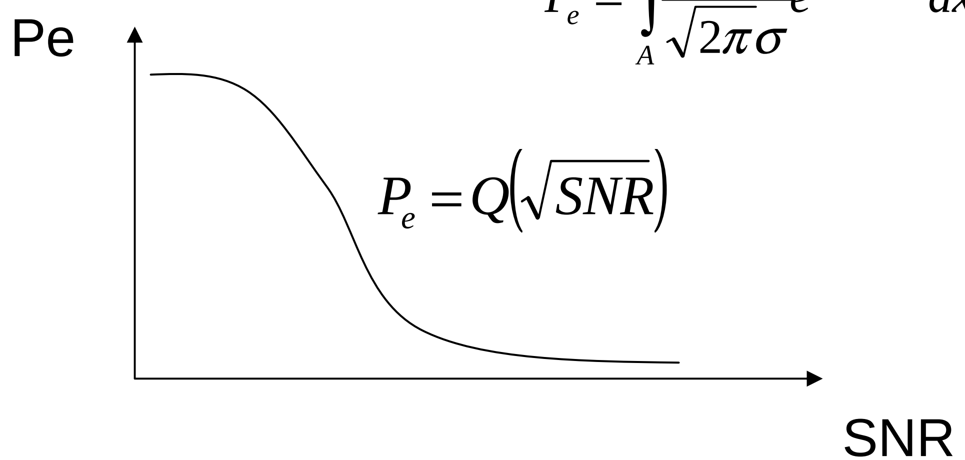
Error Probability of BPSK

- To have few errors we want A large compared to σ
- Signal to noise ratio= SNR = A^2 / σ^2
- Probability of error

$$P_e = P(A)\int_{-\infty}^{-A} \frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2} dx + P(-A)\int_{A}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2} dx$$

If $P(A) = P(-A) = 1/2$

$$P_e = \int_{A}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2} dx = \int_{A/\sigma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = Q(\sqrt{SNR})$$



Channel Error Detection

- By adding additional bits to the information bits, we can also detect errors
- Parity Check (check-sum)
  - Append a parity bit to the end of each block (called a frame) of k information bits such that the total number of '1' is even or odd
    - Used in IP packet for error detection because of simplicity
- **Example:**
    - ASCII "G" = **1 1 1 0 0 0 1**
    - with even parity = append ?
    - with odd parity = append ?

Error Detection Probability

- Parity check can detect all single bit errors in a block (if fact all error patterns with odd number of error bits)
- Cannot detect double errors or any even number of errors.
- Probability of k error bits in a n-bit frame (assuming bit error rate p_b)

$$p_n(k) = \binom{n}{k} {p_b}^k (1 - p_b)^{n-k}$$

$$\text{Ex: if } n = 10^4, p_b = 10^{-6} \text{(a good channel, e.g. ISDN)}$$

$$p_n(1) \approx 10^{-2}, p_n(2) \approx \frac{1}{2} 10^{-4} \leq p_n(1)$$

$$\text{if } n = 10^3, p_b = 10^{-3} \text{(a not so good channel, e.g. wireless)}$$

$$p_n(1) \approx 1, p_n(2) \approx \frac{1}{2}$$

Cyclic-Redundancy Codes (CRC)

General Method:

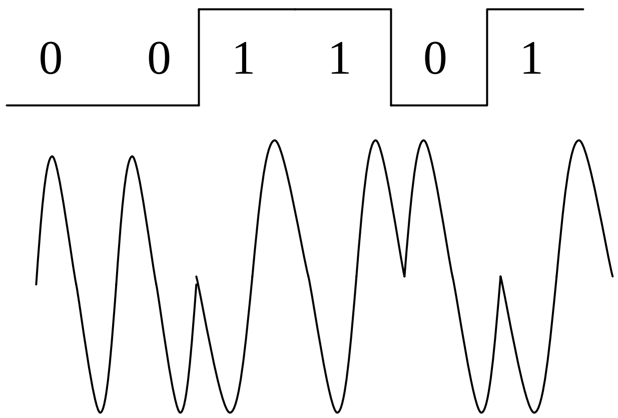
- The transmitter generates an t-bit check sequence number from a given k-bit data frame such that the resulting (k+t)-bit frame is divisible by some number
- The receiver divides the incoming frame by the same number
- If the result of the division does not leave a remainder, the receiver assumes that there was no error
- If n is large, undetectable error patterns are very unlikely
- Widely used in data communications

Error Correction by Channel Coding

- In addition to detect errors, we can correct errors by inserting parity bits. This is called channel coding.
- Simplest channel code: Repetition coding:
  - Instead of sending one 0, send three 0's.
  - If we receive more 0's than 1's: Decide 000 was sent
  - Error probability=3 or 2 bits are wrong=
    - =probability of receiving 111,110,101,011 $P_e^3 + 3(1-P_e)P_e^2 \approx 3P_e^2$
    - Smaller than Pe
    - Example: Old Pe=10^-2, New Pe=3*10^-4
  - But the information rate is decreased
    - Instead of R=1bit/sec, we have R=1/3 bits/sec
- More sophisticated channel codes can correct the same number of errors with lower redundancy (less reduction in information rate)
  - Block codes (Reed-Solomn codes), convolutional codes, turbo codes

Channel Capacity

- Channel capacity = maximum number of bits/second we can sent for reliable transmission
- What determines this?
  - The receiver typically evaluate the received signal level over entire bit interval, to determine which bit is sent (0 or 1)
  - If we sent more bits/second, the interval of each bit is shorter, channel noise will more easily make the receiver to make mistakes
  - With higher order modulation, we decode multiple bits together over each symbol interval



Noiseless Case

- To send a digital sequence, we send a sequence of pulses
  - level A representing "1", and level –A for "0"
- If the interval of each pulse is *T*, what is the maximum frequency?
  - The maximum frequency occurs when we have alternating 1 and –1, spanning 2*T* time per cycle
  - This signal has a maximum frequency of $f_{max}=1/2T$
  - With a channel with bandwidth B, $f_{max}=1/2T <= B$
    - $T>=1/2B$, $C=1/T<=2B$
    - we can send at most $C=2B$ bits/second !
- Instead of two-level pulse, we can use *N*-level pulse ($m=log_2 N$ bits/pulse), we can send $C=2Bm$ bits/second (Nyquist channel capacity)
- By increasing *N* (hence *m*), we can reach infinite bit rate!
  - What is wrong?
  - Here we assume the channel is noiseless, every level can be distinguished correctly

Channel Capacity (Noisy Case)

- When there are more levels in a pulse, the signal difference between two adjacent levels is smaller (for the same total dynamic range). Noise in communication channel is more likely to make the received/detected level differ from the actual level.

- The channel capacity depends on the signal to noise ratio (SNR)
  - SNR = signal energy / noise energy
- Shannel Channel Capacity
  - *$C=B \log_2 (1+SNR)$ bits/second*
  - *Ex: $B=6MHz$, $SNR=20$ dB = 100, $C=?$*

Advantages of Digital Communication

- More tolerant to channel noise.
  - With amplitude shift keying: as long as the noise does not change the amplitude from one level to another level, the original bits can be inferred
  - With QAM: as long as the received signal is more close to the original symbol than its neighboring symbols
  - Each repeater can regenerate the analog modulated signals from demodulated bits (noise do not accumulate)
- Can insert parity bits before sending data to allow detection/correction of errors
- Can apply digital compression techniques to reduce data rate subject to distortion criterion
- Different signals can be multiplexed more easily
  - Internet packets can contain any types of signals, cell phones can send different types of data

What Should You Know

- What is digital modulation?
- One and two dimensional modulation schemes
- Parameters of digital modulation schemes
  - Can calculate basic parameters for a given modulation scheme
  - Understand the design objective
- Channel error detection and correction
  - Should understand how simple parity check works and how repetition coding works
- Channel capacity
  - Understand the role of channel bandwidth and SNR in determining channel capacity
- Advantages of digital communications

References

- A. Leon-Garcia, I. Widjala, Communication networks, Chap 3: Digital transmission fundamentals.