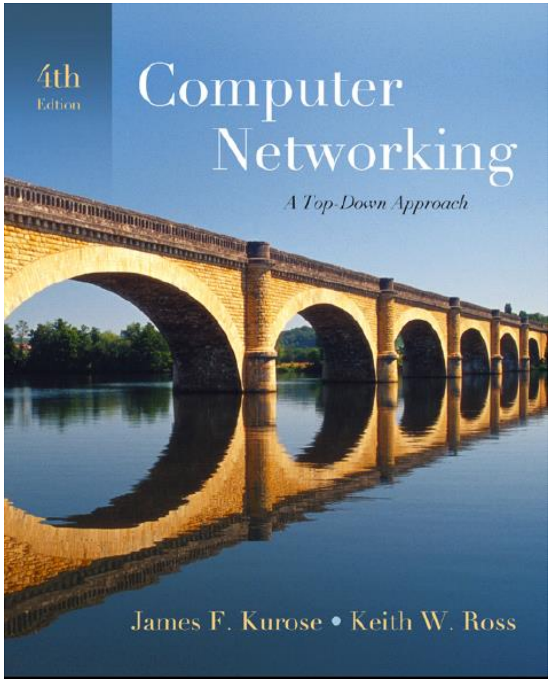
Proportional Fairness



Computer Networking: A Top Down Approach
4^{th} edition.
Jim Kurose, Keith Ross
Addison-Wesley, July 2007.

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2: Optimization-based approach towards congestion control

Resource allocation as optimization problem:

- how to allocate resources (e.g., bandwidth) to optimize some objective function
- maybe not possible that optimality exactly obtained but…
  - optimization framework as means to explicitly steer network towards desirable operating point
  - practical congestion control as distributed asynchronous implementations of optimization algorithm
  - systematic approach towards protocol design

Proportional Fairness

- Vector of rates, $\{X_s\}$, proportionally fair if feasible and for any other feasible vector $\{X_s^*\}$:

$$\sum_{s \in S} \frac{x_s^* - x_s}{x_s} \le 0$$

- Optimal Solution is derived by

$$\max \sum \log(X_s)$$

subject to capacity constraints

- GDS: $$\sum_{s \in S} \frac{x_s^* - x_s}{x_s} = \sum_{s \in S} \frac{\Delta x_s}{x_s} \approx \sum_{s \in S} \Delta(\log x_s) \le 0$$

Implies that $\sum \log(X_s)$ is the maximum possible - Also, this allocation is "fair" in the sense that in order to deviate from it, the total percentage of deviations has to be non-positive, meaning that small flows have to "suffer" relatively more

Optimization Problem

$$\max_{x_s \geq 0} \quad \sum_s U_s(x_s)$$

$$\text{subject to} \quad \sum_{s \in S(l)} x_s \leq c_l, \forall l \in L$$

"system" problem

- maximize system utility (note: all sources equal)
- constraint: bandwidth used less than capacity
- centralized solution to optimization impractical
  - must know all utility functions
  - impractical for large number of sources
  - we'll see: congestion control as distributed asynchronous algorithms to solve this problem

The user view

- user can choose amount to pay per unit time, w_s
- Would like allocated bandwidth, x_s in proportion to w_s

$$x_s = \frac{w_s}{p_s}$$

- p_s could be viewed as charge per unit flow for user s

$$\max \quad U_s\left(\frac{w_s}{p_s}\right) - w_s$$

$$\text{subject to} \quad w_s \geq 0$$

user problem

user's utility

cost

The network view

- suppose network knows vector $\{w_s\}$, chosen by users
- network wants to maximize logarithmic utility function

$$\max_{x_s \geq 0} \quad \sum_s w_s \log x_s$$

network problem

$$\text{subject to} \quad \sum_{s \in S(l)} x_s \leq c_l$$

Solution existence

- There exist prices, p_s, source rates, x_s, and amount-to-pay-per-unit-time, $w_s = p_s x_s$ such that
  - $\{W_s\}$ solves user problem
  - $\{X_s\}$ solves the network problem
  - $\{X_s\}$ is the unique solution to the system problem
  - p_s = Sum of links' shadow prices over the path of s

$$\max \quad U_s\left(\frac{w_s}{p_s}\right) - w_s$$

$$\text{subject to} \quad w_s \geq 0$$

$$\max_{x_s \geq 0} \quad \sum_s w_s \log x_s$$

$$\text{subject to} \quad \sum_{s \in S(l)} x_s \leq c_l$$

$$\max_{x_s \geq 0} \quad \sum_s U_s(x_s)$$

$$\text{subject to} \quad \sum_{s \in S(l)} x_s \leq c_l, \forall l \in L$$

Proportional Fairness Property

- If w_r=1, then $\{X_s\}$ solves the network problem IFF it is proportionally fair
- Related results exist for the case that w_r not equal 1.
- GDS: Solution of the weighted proportional fairness for the derived w_r is an additional property of the socially optimal solution

Solving the network problem

- Results so far: *existence* - solution exists with given properties
- How to *compute* solution?
  - ideally: distributed solution easily embodied in protocol
  - insight into existing protocol

Outline of Approach (GDS):

- Tatonnement process in order to reach equilibrium prices in bandwidth market:
  - Increase (resp. decrease) bandwidth per user if his willingness-to-pay buys him more (resp. less) with the present prices
    - How?? → next slide
  - Increase (resp. decrease) unit-bandwidth price per link if total allocated bandwidth is lower (resp. higher) than link capacity
    - How?? → next slide

Solving the network problem

$$\frac{d}{dt}x_s(t) = k\left(w_s - x_s(t)\sum_{l\in L(s)} p_l(t)\right)$$

change in bandwidth allocation at s — linear increase — multiplicative decrease

where $p_l(t) = g_l\left(\sum_{l\in L(s)} x_s(t)\right)$

congestion "signal": function of aggregate rate at link l, fed back to s.

Solving the network problem

$$\frac{d}{dt}x_s(t) = k\left(w_s - x_s(t)\sum_{l \in L(s)} p_l(t)\right)$$

- Results:
  - converges to solution of "relaxation" of network problem
  - $x_s(t)\Sigma p_l(t)$ converges to w_s
- Interpretation: TCP-like algorithm to iteratively solve optimal rate allocation!
- GDS: In TCP the two phases alternate, while in the above mechanism they are both applied at the same time

Optimization-based congestion control: summary

- bandwidth allocation as optimization problem:
- practical congestion control (TCP) as distributed asynchronous implementations of optimization algorithm
- optimization framework as means to explicitly steer network towards desirable operating point
- systematic approach towards protocol design