Proportional Fairness

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2: Optimization-based approach towards congestion control

Resource allocation as optimization problem:
- how to allocate resources (e.g., bandwidth) to optimize some objective function
- maybe not possible that optimality exactly obtained but...
  - optimization framework as means to explicitly steer network towards desirable operating point
  - practical congestion control as distributed asynchronous implementations of optimization algorithm
  - systematic approach towards protocol design
Proportional Fairness

- Vector of rates, \( \{X_s\} \), proportionally fair if feasible and for any other feasible vector \( \{X_s^*\} \):
  \[
  \sum_{s \in S} \frac{x_s^* - x_s}{x_s} \leq 0
  \]

- Optimal Solution is derived by
  \[
  \max \sum \log(X_s) \\
  \text{subject to capacity constraints}
  \]

- GDS:
  \[
  \sum_{s \in S} \frac{x_s^* - x_s}{x_s} = \sum_{s \in S} \frac{\Delta x_s}{x_s} \approx \sum_{s \in S} \Delta(\log x_s) \leq 0
  \]

Implies that \( \sum \log(X_s) \) is the maximum possible - Also, this allocation is “fair” in the sense that in order to deviate from it, the total percentage of deviations has to be non-positive, meaning that small flows have to “suffer” relatively more
Optimization Problem

max \sum_{s} U_{s}(x_{s})

subject to \sum_{s \in S(l)} x_{s} \leq c_{l}, \forall l \in L

- maximize system utility (note: all sources equal)
- constraint: bandwidth used less than capacity
- centralized solution to optimization impractical
  - must know all utility functions
  - impractical for large number of sources
  - we’ll see: congestion control as distributed asynchronous algorithms to solve this problem

“system” problem
The user view

- user can choose amount to pay per unit time, $w_s$
- Would like allocated bandwidth, $x_s$ in proportion to $w_s$

$$x_s = \frac{w_s}{p_s}$$

- $p_s$ could be viewed as charge per unit flow for user $s$

$$\text{max} \quad U_s \left( \frac{w_s}{p_s} \right) - w_s$$

subject to $w_s \geq 0$

user's utility

user problem

cost
The network view

- suppose network knows vector \( \{w_s\} \), chosen by users
- network wants to maximize logarithmic utility function

\[
\begin{align*}
\max_{x_s \geq 0} & \quad \sum_{s} w_s \log x_s \\
\text{subject to} & \quad \sum_{s \in S(l)} x_s \leq c_l
\end{align*}
\]

network problem
Solution existence

- There exist prices, $p_s$, source rates, $x_s$, and amount-to-pay-per-unit-time, $w_s = p_s x_s$ such that
  - $\{W_s\}$ solves user problem
  - $\{X_s\}$ solves the network problem
  - $\{X_s\}$ is the unique solution to the system problem
  - $p_s = \text{Sum of links' shadow prices over the path of } s$

\[
\max \quad U_s \left( \frac{w_s}{p_s} \right) - w_s
\]
subject to $w_s \geq 0$

\[
\max \quad \sum_s w_s \log x_s
\]
subject to $\sum_{s \in S(l)} x_s \leq c_l$

\[
\max \quad \sum_s U_s(x_s)
\]
subject to $\sum_{s \in S(l)} x_s \leq c_l, \forall l \in L$
Proportional Fairness Property

- If \(\omega_r = 1\), then \(\{X_s\}\) solves the network problem IFF it is proportionally fair.

- Related results exist for the case that \(\omega_r\) not equal 1.

- GDS: Solution of the weighted proportional fairness for the derived \(\omega_r\) is an additional property of the socially optimal solution.
Solving the network problem

- Results so far: *existence* - solution exists with given properties
- How to *compute* solution?
  - ideally: distributed solution easily embodied in protocol
  - insight into existing protocol
Outline of Approach (GDS):

- Tatonnement process in order to reach equilibrium prices in bandwidth market:
  - Increase (resp. decrease) bandwidth per user if his willingness-to-pay buys him more (resp. less) with the present prices
    - How?? → next slide
  - Increase (resp. decrease) unit-bandwidth price per link if total allocated bandwidth is lower (resp. higher) than link capacity
    - How?? → next slide
Solving the network problem

\[ \frac{d}{dt} x_s(t) = k \left( w_s - x_s(t) \sum_{l \in L(s)} p_l(t) \right) \]

- change in bandwidth allocation at \( s \)
- linear increase
- multiplicative decrease

where \( p_l(t) = g_l \left( \sum_{l \in L(s)} x_s(t) \right) \)

congestion “signal”: function of aggregate rate at link \( l \), fed back to \( s \).
Solving the network problem

\[ \frac{d}{dt} x_s(t) = k \left( w_s - x_s(t) \sum_{l \in L(s)} p_l(t) \right) \]

- **Results:**
  - \( x_s(t) \) converges to solution of “relaxation” of network problem
  - \( x_s(t) \sum p_l(t) \) converges to \( w_s \)

- **Interpretation:** TCP-like algorithm to iteratively solve optimal rate allocation!

- **GDS:** In TCP the two phases alternate, while in the above mechanism they are both applied at the same time
Optimization-based congestion control: summary

- bandwidth allocation as optimization problem:
- practical congestion control (TCP) as distributed asynchronous implementations of optimization algorithm
- optimization framework as means to explicitly steer network towards desirable operating point
- systematic approach towards protocol design