Max-Min Fairness vs Proportional Fairness

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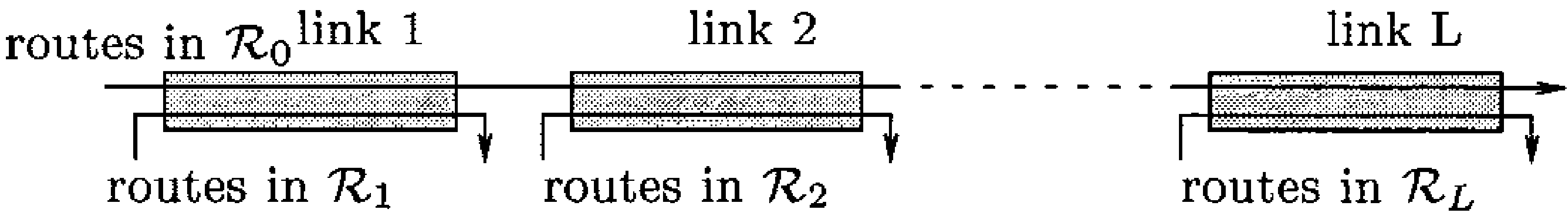
Linear Network (Massoulie & Roberts)

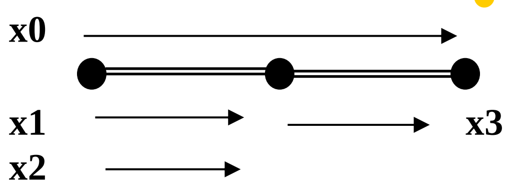
- Unit capacity in all links
- n_0 → number of flows in "long" route in R_0
- n_i → number of flows in "short" route (hop i) R_i



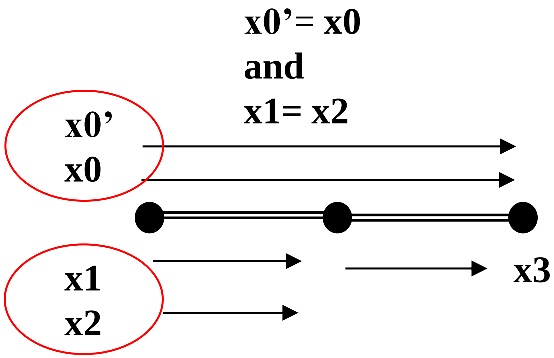
Comparison of Max-min Fair and Proportionally Fair Allocations

- Assume that $n_0 = n_i = 1$
- Max-min fair allocation:
  - $x_0 = x_i = ½$
  - Total throughput = (L+1)/2
- Proportionally fair allocation (see slide 5):
  - $x_0 = 1/(L+1)$
  - $x_i = L/(L+1)$
  - The "long" flow is "penalized" for requiring resources in multiple hops
  - Total throughput = L – (L-1)/(L+1), approaches L-1 for large L
  - The maximum total throughput is L; attained if the "long" flow is throttled ($x_0 = 0$)

Simplifying the Calculation of the Proportionally Fair Allocation



- Assume both links have capacity 1
- $x_0 + x_3 \leq 1$. Since we aim to maximize the sum $\log(x_0) + \log(x_1) + \log(x_2) + \log(x_3)$, for a given x_0 we should take $x_3 = 1 - x_0$ due the fact that the logarithmic function is <u>increasing</u>
- By a similar argument, we should take $x_1 + x_2 = 1 - x_0$. Due to <u>concavity</u> of the logarithmic function, when $x_1 + x_2 = a$, then $\log(x_1) + \log(x_2)$ is maximized for $x_1 = x_2 = a/2$. Thus, we should take $x_1 = x_2 = (1 - x_0)/2$
- Now we are left with the equivalent problem of maximizing $\log(x_0) + \log(1-x_0) + 2\log[(1-x_0)/2]$ which can be solved easily.



- In general, due to concavity, in the proportionally fair allocation flows having the same path are equal, and this leads to simplification of the maximization problem

Derivation of the Proportionally Fair Allocations of slide 3

- Apply previous methodology for $n_0 = n_i = 1$
- Maximize the sum $\log(x_0) + \log(x_1) + \ldots + \log(x_L)$,

  $x_i = 1 - x_0$ -->

- Max $[\log(x_0) + L * \log(1-x_0)]$. Differentiating the objective function wrt x_0 gives:

  $1/x_0 - L/(1-x_0) = 0$

- Solving for x_0 gives $x_0 = 1/(L+1)$
- Then $x_i = 1 - x_0 = L/(L+1)$