User Utility Function:

1. **Main Definitions**
2. **Utility for Bandwidth**

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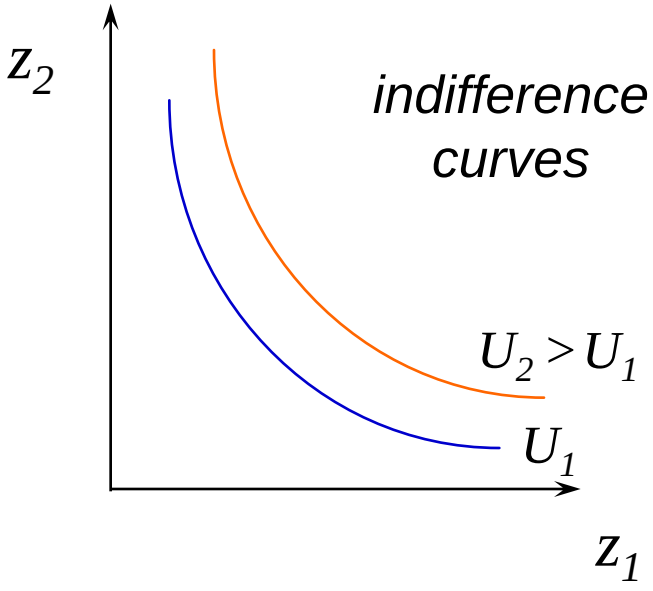
The Notion of User Utility Function

- A user has to select the quantities
  $\boldsymbol{z} = (z_1,..,z_n)$ of n goods.
  - A choice $\boldsymbol{z}$ is preferred to $\boldsymbol{z}'$ by this user and only if
    $U(\boldsymbol{z}) > U(\boldsymbol{z}')$



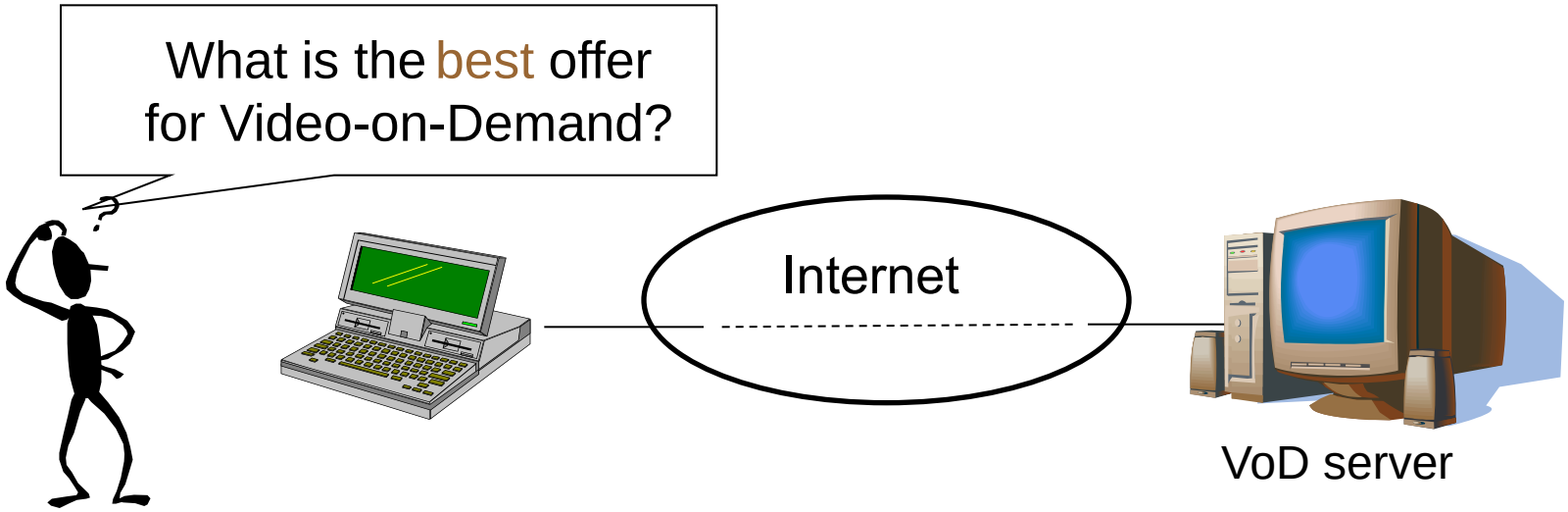
Properties

- Utility function defines a complete ordering, but
- … it is not uniquely defined: A monotonical transform of $U(\boldsymbol{z})$ expresses the same preferences; e.g. log $U(\boldsymbol{z})$
- Unique definition:
  Utility $U(\boldsymbol{z})$ = amount of money the user is willing to pay for $\boldsymbol{z}$
  - Provides a measure of:
    - the user's satisfaction from using the vector $\boldsymbol{z}$ of goods, "translated" in monetary units
    - the amount of money the user will earn from reselling $\boldsymbol{z}$

The User Problem in Networks

- User runs several applications
- Quality of Service (QoS) influences the acquired satisfaction of the user per application
  - Payment may also depend on QoS level

➔The user should make the best choice of QoS levels



For the user to make the best selection → We need …

1. A user utility function to evaluate the offers:
   - goods ↔ flows for applications
     - Or connections, depending on the technology
   - quantities ↔ QoS levels of flows
     → use bandwidth as a proxy for QoS

2. The user's optimization criterion:
   - *Maximize utility*
   - *Minimize charge*
   - *Maximize net benefit*
     ⇔ *Maximize (utility - charge)*

Motivating Net Benefit Maximization (I)

- A user has:
  - To select the vector of quantities $\boldsymbol{z}$ of goods
  - Without exceeding his fixed budget B.

- The user benefits from both:
  - the goods he will use → $U(\boldsymbol{z})$
  - the amount of money he will save → $B - c(\boldsymbol{z})$

→ Quasi-linear total utility:

$$U(\boldsymbol{z}) + B - c(\boldsymbol{z})$$

  - = total utility in case of resale of the goods
    - ☒ the net profit from the resale equals $U(\boldsymbol{z}) - c(\boldsymbol{z})$

Motivating Net Benefit Maximization

→ Quasi-linear total utility:

$$U(\mathbf{z}) + B - c(\mathbf{z})$$

- User Problem: Select $\mathbf{z}$ so as to

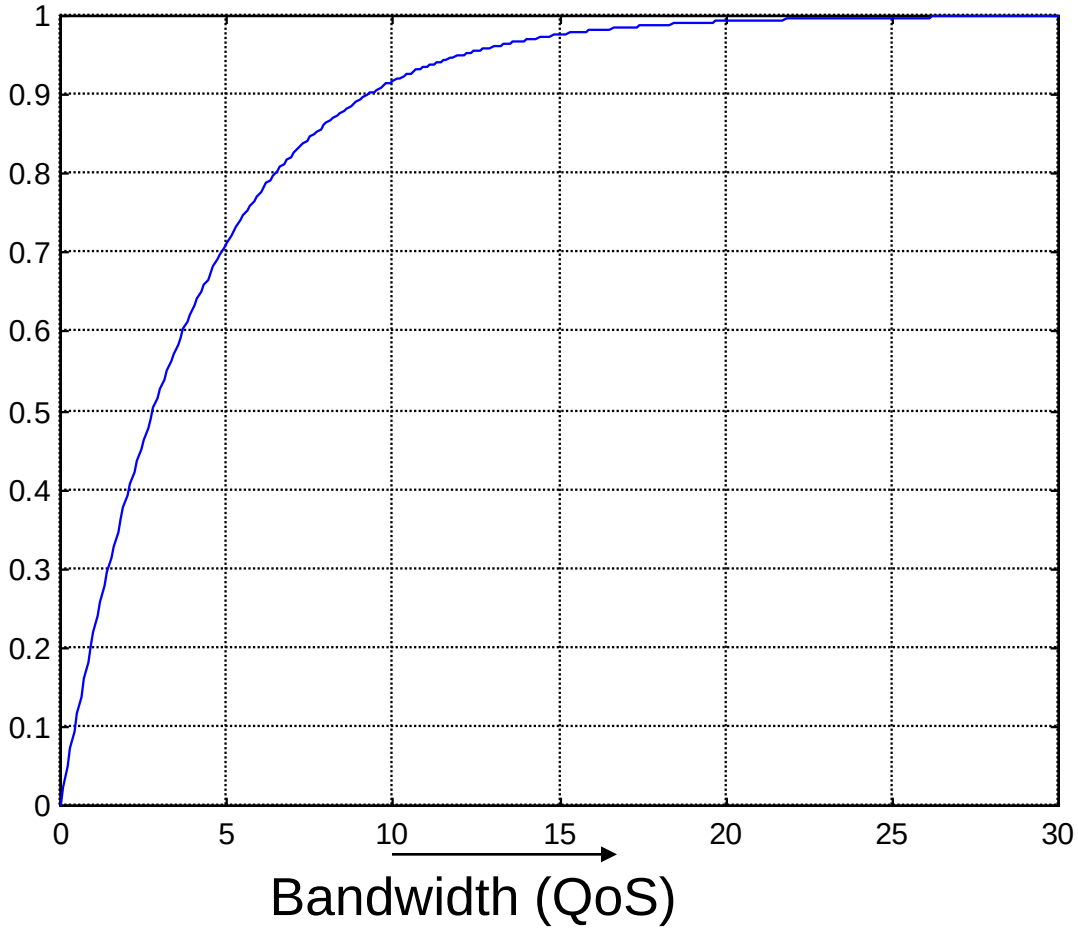
$$\text{Max}\{U(\mathbf{z}) + B - c(\mathbf{z})\} \Leftrightarrow \text{Max}\{U(\mathbf{z}) - c(\mathbf{z})\} + B$$
$$\text{s.t.} \quad c(\mathbf{z}) \leq B$$

- "Almost" equivalent to unconstrained

$$\text{Max}_{\mathbf{z}}\{U(\mathbf{z}) - c(\mathbf{z})\}$$

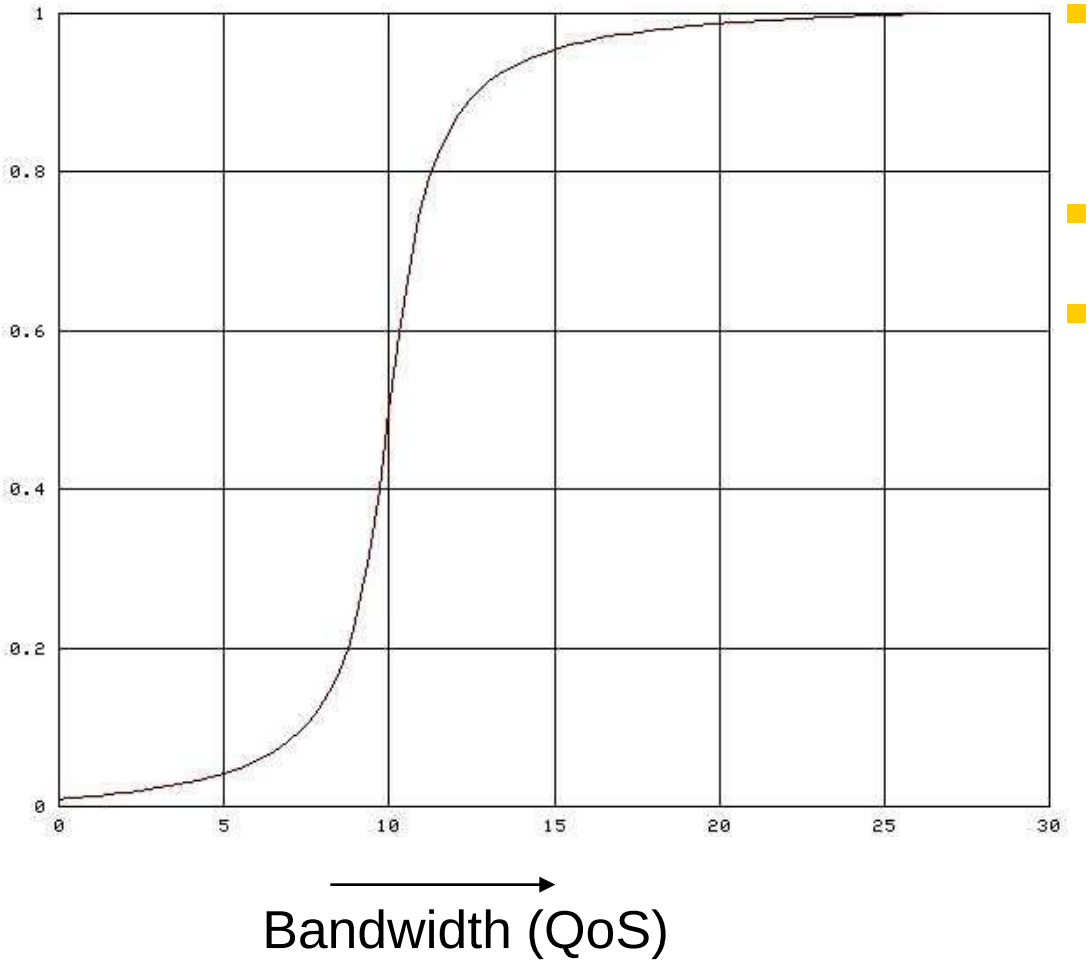
because the optimal usually does not exhaust the budget

Utility for Elastic Services



- FTP, web-browsing
- Concave utility function
- Each unit of extra bandwidth is valuable, but the return (=extra utility) is diminishing as the already acquired amount increases
- E.g. $U(x) = x^{1/2}$

Utility for Guaranteed Service



- Multimedia
- sigmoid utility function
- Expresses the requirement for a minimum or even a fixed bandwidth amount
  - Less bandwidth is almost useless
  - more has little extra value