User Utility Function:

1. Main Definitions
2. Utility for Bandwidth

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The Notion of User Utility Function

- A user has to select the quantities $z = (z_1, \ldots, z_n)$ of $n$ goods.
  - A choice $z$ is preferred to $z'$ by this user and only if $U(z) > U(z')$
Properties

• Utility function defines a complete ordering, but
• ... it is not uniquely defined: A monotonical transform of $U(z)$ expresses the same preferences; e.g. $\log U(z)$

• Unique definition:
Utility $U(z) = \text{amount of money}$ the user is willing to pay for $z$
  ▪ Provides a measure of:
    ➢ the user’s satisfaction from using the vector $z$ of goods, “translated” in monetary units
    ➢ the amount of money the user will earn from reselling $z$
The User Problem in Networks

- User runs several applications
- Quality of Service (QoS) influences the acquired satisfaction of the user per application
  - Payment may also depend on QoS level

⇒ The user should make the best choice of QoS levels

What is the best offer for Video-on-Demand?
For the user to make the best selection
→ We need ...

1. A user utility function to evaluate the offers:
   - goods ↔ flows for applications
   - Or connections, depending on the technology
   - quantities ↔ QoS levels of flows
     → use bandwidth as a proxy for QoS

2. The user’s optimization criterion:
   - Maximize utility
   - Minimize charge
   - Maximize net benefit
     ⇔ Maximize (utility - charge)
Motivating Net Benefit Maximization (I)

- A user has:
  - To select the vector of quantities $z$ of goods
  - Without exceeding his fixed budget $B$.

- The user benefits from both:
  - the goods he will use $\to U(z)$
  - the amount of money he will save $\to B - c(z)$

$\Rightarrow$ Quasi-linear total utility:

$$U(z) + B - c(z)$$

- = total utility in case of resale of the goods
  - the net profit from the resale equals $U(z) - c(z)$
Motivating Net Benefit Maximization

→ Quasi-linear total utility:

\[ U(z) + B - c(z) \]

- User Problem: Select \( z \) so as to

\[
\text{Max}\{U(z) + B - c(z)\} \iff \text{Max}\{U(z) - c(z)\} + B
\]

\[ \text{s.t. } c(z) \leq B \]

- “Almost” equivalent to unconstrained

\[
\text{Max}_z\{U(z) - c(z)\}
\]

because the optimal usually does not exhaust the budget
Utility for Elastic Services

- FTP, web-browsing
- Concave utility function
- Each unit of extra bandwidth is valuable, but the return (=extra utility) is diminishing as the already acquired amount increases
- E.g. $U(x) = x^{1/2}$
Utility for Guaranteed Service

- Multimedia
- **sigmoid** utility function
- Expresses the requirement for a minimum or even a fixed bandwidth amount
  - Less bandwidth is almost useless
  - more has little extra value

Bandwidth (QoS)