Topics on Routing

George D. Stamoulis
AUEB

1) The Bellman-Ford algorithm as the derivation of a fixed-point

Bellman-Ford Algorithm

- Compute the shortest distances D_i to a particular node 0 (=«destination») from all other nodes i
- $D_i = \min_{j\in N(i)}\{c_{ij} + D_j\}$ όπου $N(i)$ το σύνολο γειτόνων του i
- Writing these equations for all nodes i, we attain a system of n equations with n unknowns – the system is of the "fixed point" type, and can be solved iteratively

$$D_i^{(k+1)} = \min_{j\in N(i)}\{c_{ij} + D_j^{(k)}\}$$

- Convergence of this iteration is attained under general conditions
- Similar properties apply for the computation of the shortest distances D_i to a particular node 0 (=«destination») to all other nodes i
  - For particular initial values, the iteration terminates in a finite number of steps → slide 7.

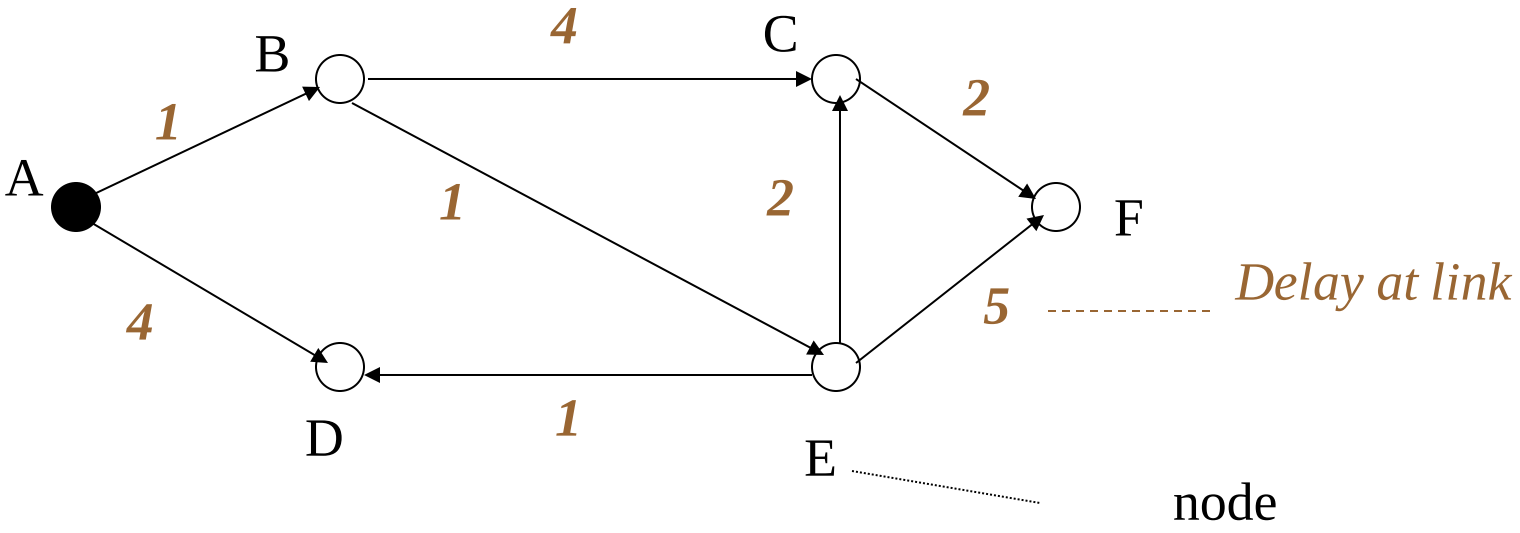
Fixed point problems

- Equations of the type $x=f(x)$, where f is a continuous function
  → e.g. $x=1/3*e^x$ - the solution is x ~1.5
- Iterative solution $x_{k+1}= f(x_k)$
  - If x_k converges to a limit x^*, then x^* is the root (solution) of the equation, because $f(x^*) = x^*$ since f is a continuous function
  - Termination applies if $x_{n+1}=x_n$. Then, $x_n=f(x_n)$, which implies that x_n is the root
- Convergence is not guaranteed
  Equations $x=2x$ and $x=x/2$ both have 0 as a root
  a. $x_{k+1}=2x_k$ and for $x_0=1$ → $x_k=2^k$ does not converge!
  b. $x_{k+1}=x_k/2$ and for $x_0=1$ → $x_k=1/2^k$ converges to 0.
- Similar properties apply for the vector equation **x=g**(**x**), that is for a system of equations

2) Example of application of Bellman-Ford and Dijkstra algorithms

Problem Definition

- Derivation of shortest paths from node A to all other nodes



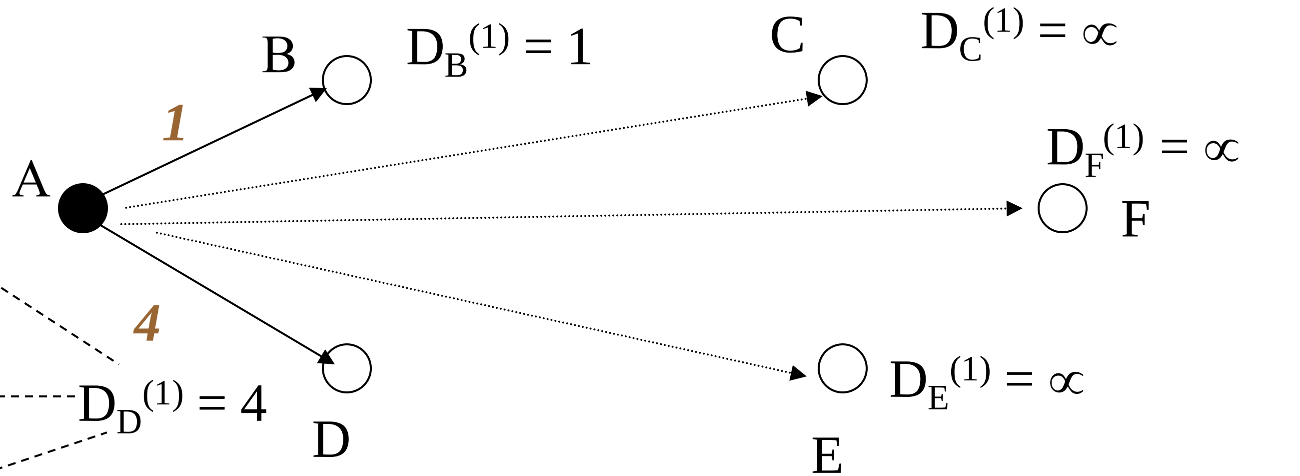
Bellman-Ford algorithm: Steps (I)

- Step 1

Index of Step

Current estimate of distance from node A to destination node

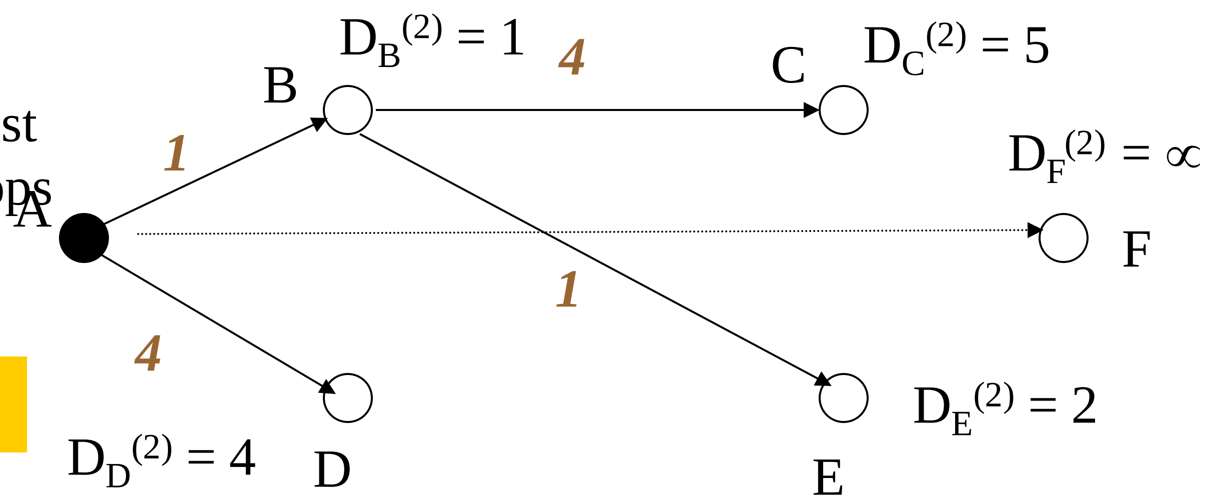
Index of destination node



Total length (delay) of shortest paths to i but with k or less hops

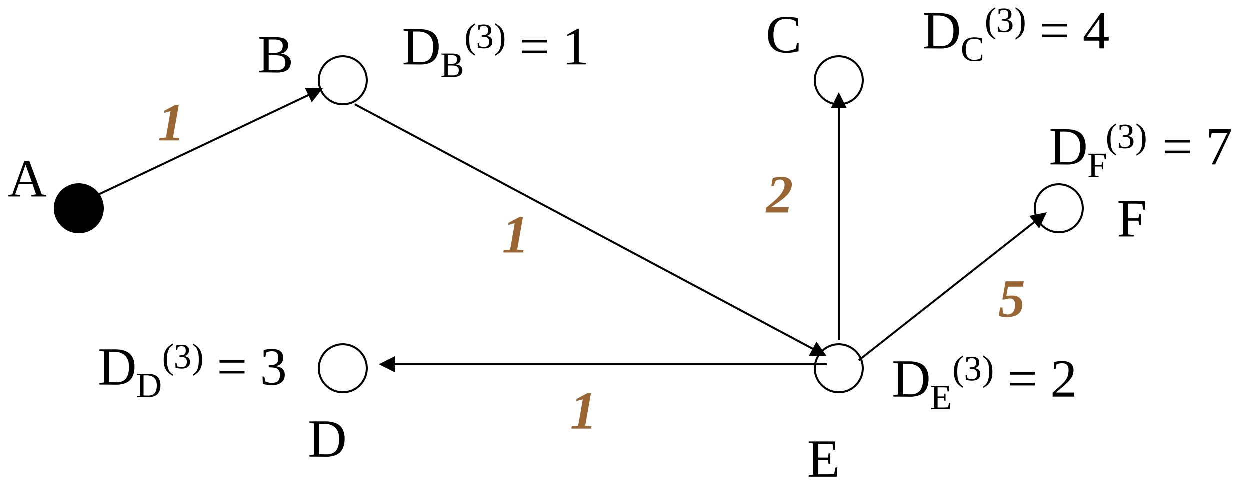
- Step 2

$$D_i^{(k)} = \min_j\{D_j^{(k-1)} + c_{ji}\}$$

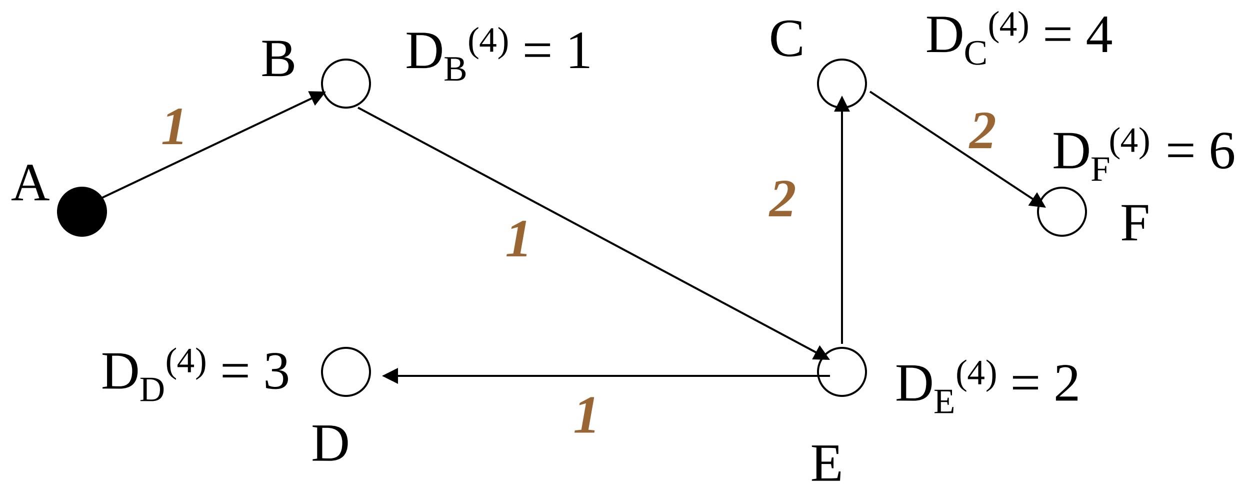


Bellman-Ford algorithm: Steps (II)

- Step 3



- Step 4 (Final)

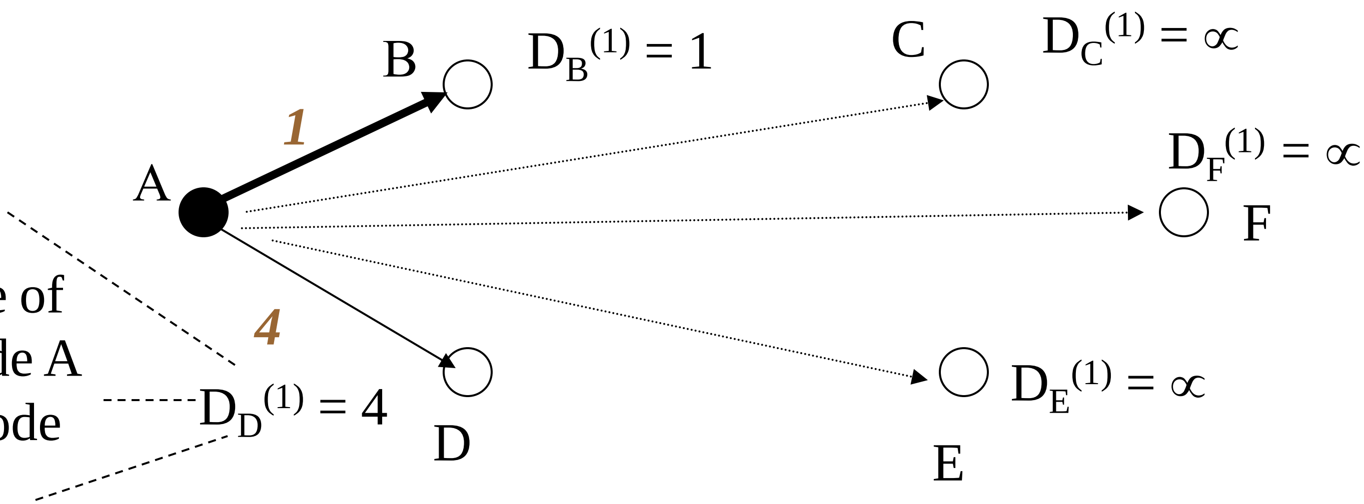


Dijkstra algorithm: Steps (I)

- Βήμα 1

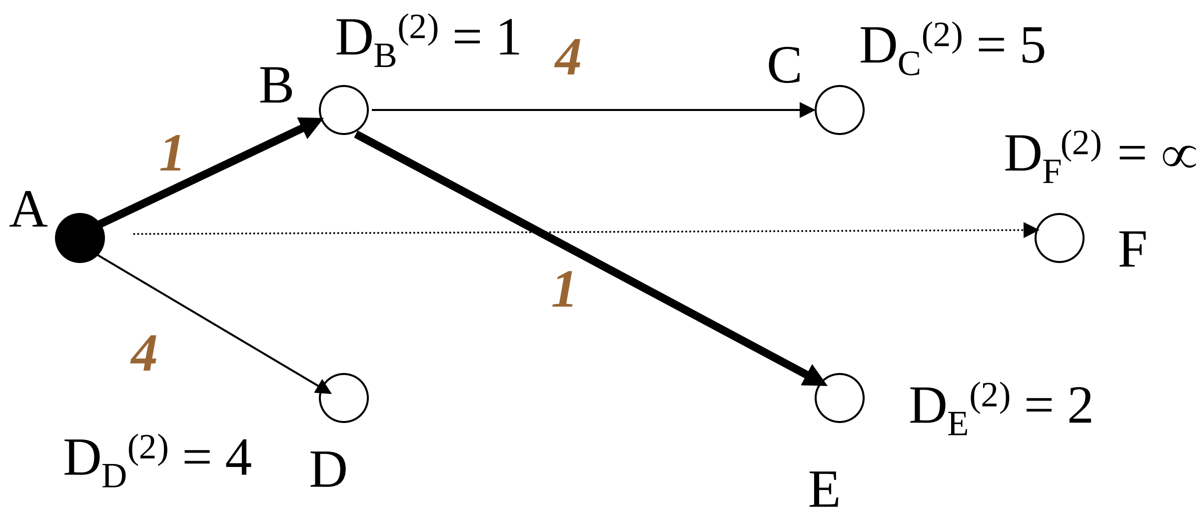
Index of Step

Current estimate of distance from node A to destination node



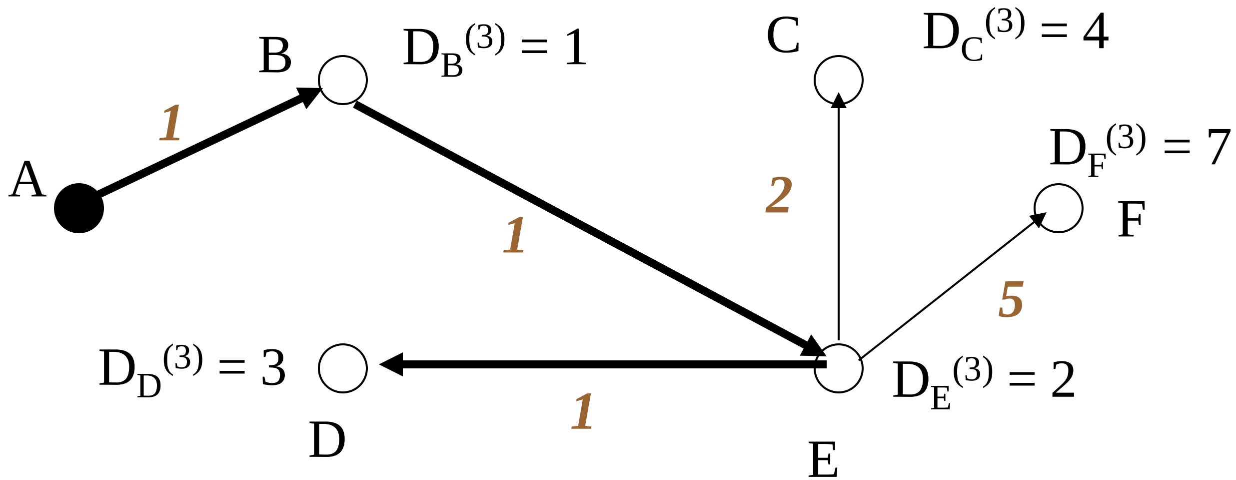
Index of destination node

- Step 2

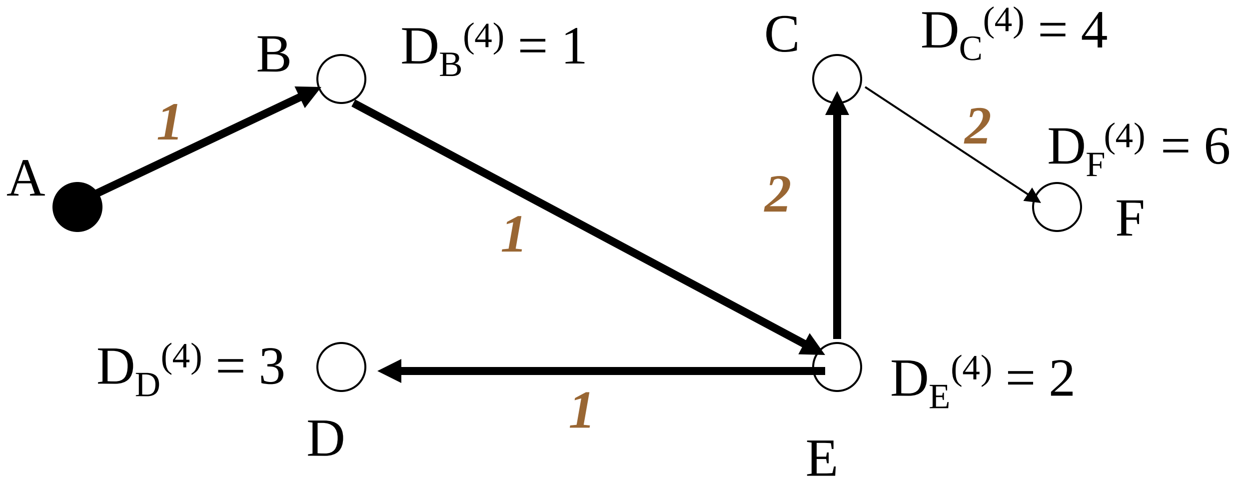


Dijkstra algorithm: Steps (II)

- Step 3

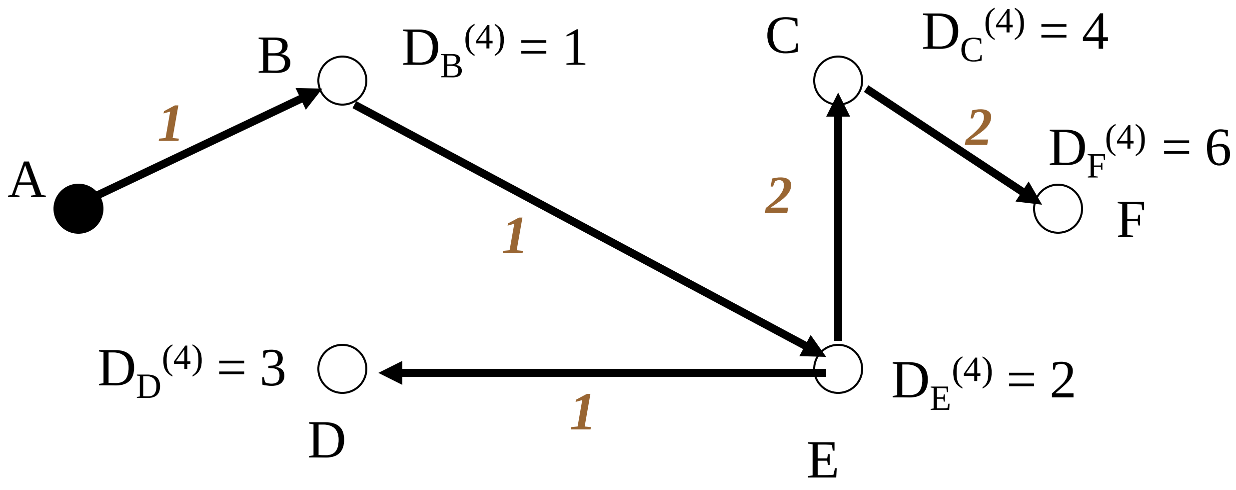


- Step 4



Dijkstra algorithm: Steps (III)

- Step 5 (Final)



3) Evolution of routing protocols employed in the Internet

1st Generation - 1969

- Based on Bellman-Ford algorithm
- Metric: estimate of delay per link
  - based on queue size, without use of link rate
- Distributed implementation
- Modification of routing tables upon reception of new delay estimates
- Slow reaction to changes of the congestion levels

2nd Generation - 1979

- Based on Dijkstra algorithm
- Metric: estimate of delay per link
  - measured directly
- Good performance for medium and light network loads
- Discrepancy between estimated and observed delays at paths for high network loads

3^{rd} Generation - 1987

- Modification of metric's estimation:
  - Average delay in intervals of 10 sec
  - Combination with previous estimates

- Finally: Hierarchical routing, using BGP at the "core" of Internet
  - and RIP with path delay equal to number of hops