Convergence of MCMC and Loopy BP in the Tree Uniqueness Region for the Hard-Core Model

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joint work with:

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$Z = Z(G, \lambda)$ is the *partition function*.

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- computationally *hard problem* [Valiant 1979]
- focus on the *approximation algorithms*

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For T and λ

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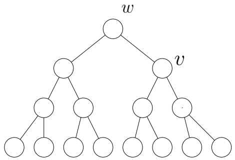

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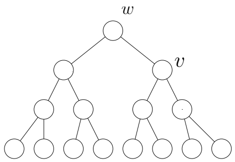

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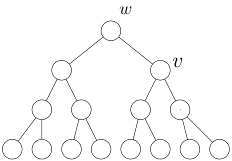

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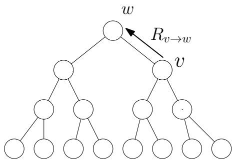

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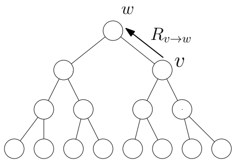

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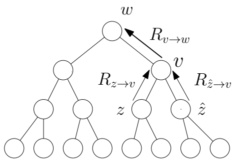

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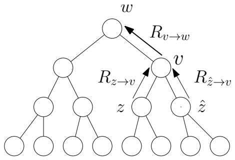

Start from arbitrary $R^0_{v\to w}$s,
iterate like

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Convergence on trees

There exists i_0 such that for every $i \geq i_0$ and every $(R^0_{v \to w})_{\{v,w\} \in E}$ we have

$$R^i_{v \to w} = R^*_{v \to w}$$

In turn

$$\mu(v \text{ occupied} | w \text{ unoccupied}) = q^* = \frac{R^*_{v \to w}}{1 + R^*_{v \to w}}$$

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BP is an elaborate version of *Dynamic Programing*

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$$\Pr[\hat{Z} \in (1 \pm \epsilon) Z(G, \lambda)] \geq 1 - \delta$$

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Intuition

... the larger λ the harder is to approximate $Z(G, \lambda)$

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For triangle-free Δ-regular graphs, where $\Delta \geq 3$, and for all $\lambda > \lambda_c(\Delta)$, it is NP-hard to approximate the partition function within factor $e^{\Theta(n)}$.

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- Galanis, Ge, Stefankovic, Vigoda, Yang (2011)
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What is $\lambda_c(\Delta)$? [Kelly 1985]

Critical point for "uniqueness/non-uniqueness" transition of the hard-core model on Δ regular trees

$$\lambda_c(\Delta) := \frac{(\Delta - 1)^{\Delta - 1}}{(\Delta - 2)^{\Delta}} \sim \frac{e}{\Delta}$$

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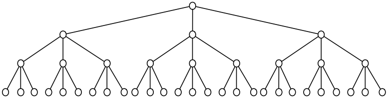

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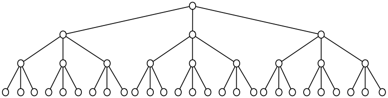

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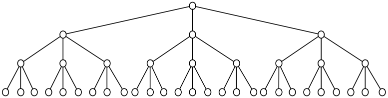

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$\lambda \leq \lambda_c(\Delta)$: No boundary effects root.

$\lambda > \lambda_c(\Delta)$: Exist boundaries effect root.

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  - this step requires $\lambda < \lambda_c$

Performance Weitz's algorithm

Approximation guarantees

For all $\delta > 0$, there exists constant $C = C(\delta) > 0$, for all Δ all G of maximum degree Δ, all $\lambda < (1-\delta)\lambda_c(\Delta)$ all $\epsilon > 0$ Weitz's algorithm returns an estimation $\hat{Z}$ of the partition function $Z(G,\lambda)$ such that

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in time $O((n/\epsilon)^{C \log \Delta})$.

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- Li, Lu, and Yin (2012), (2013)
- Restrepo, Shin, Tetali, Vigoda and Yang (2013)
- Sinclair, Srivastava and Yin (2013)

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it is desirable that the chain mixes "fast"

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The chain converges to the hard-core model with fugacity λ.

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Theorem

For all $\delta > 0$, *there exists* $\Delta_0 = \Delta_0(\delta)$ *for all graphs* $G = (V, E)$ *of maximum degree* $\Delta \geq \Delta_0$ *and* girth ≥ 7, *all* $\lambda < (1 - \delta)\lambda_c(\Delta)$, *the mixing time of the Glauber dynamics satisfies*

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Mixing Time ...

$$T_{mix} = \min\{t : \text{ for all } X_0, d_{tv}(X_t, \mu) \leq 1/4\},$$

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Previous work

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- Dyer Greenhill, Luby, Vigoda

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Mossel, Weitz, Wormald (2009)

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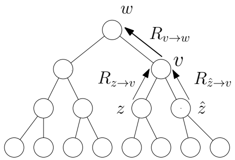

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- . . . if does, we do not know where exactly it converges

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we also have convergence for the BP estimate of $\mu(v \text{ is occupied})$

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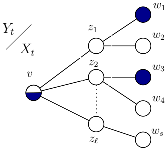

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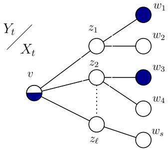
Consider copies $(X_s), (Y_s)$ such that $X_t \oplus Y_t = \{v\}$

$$\mathbb{E}\left[\Phi(X_{t+1}, Y_{t+1}) \middle| X_t, Y_t\right] \leq (1-\gamma)\Phi(X_t, Y_t).$$



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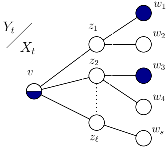

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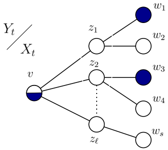

Path Coupling Example

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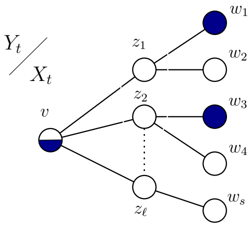
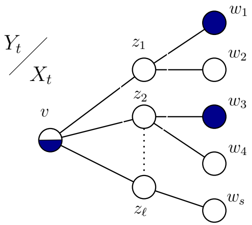


Path Coupling Example

Expected distance

$$\mathbb{E}\left[\Phi(X_{t+1}, Y_{t+1}) \middle| X_t, Y_t\right] = \left(1 - \frac{1}{n}\right)\Phi_v + \sum_{z_i} \Pr[z_i \in Y_{t+1}] \cdot \Phi_{z_i}$$



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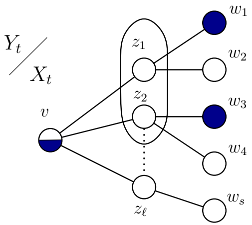

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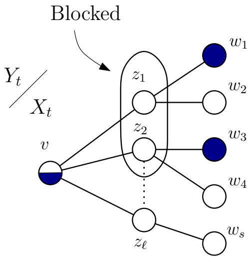

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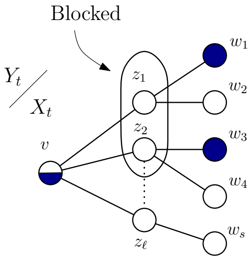
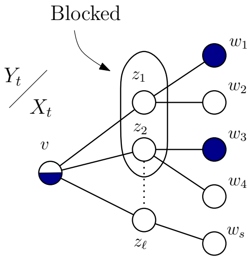


Path Coupling Example

Path coupling condition

$$\Phi_v > \frac{\lambda}{1+\lambda} \sum_{z_i} \mathbf{1}\{z_i \text{ unblocked in } Y_t\} \cdot \Phi_{z_i}$$



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  - dynamics gets *local uniformity*
- Given Φ and convergence of Glauber dynamics we show rapid mixing

Unblocked Neighbors and loopy BP

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- $\omega^*(z) \approx \mu(z \text{ is unblocked})$

Back to Path Coupling

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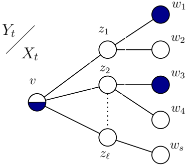

Back to Path Coupling

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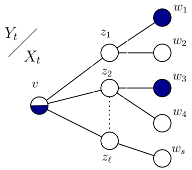

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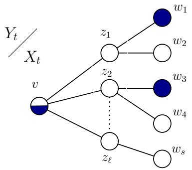

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There is Φ such that

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$n \times n$ matrix $\mathcal{C}$

$$\mathcal{C}(v,z) = \begin{cases} \frac{\lambda\omega^*(z)}{1+\lambda\omega^*(z)} & \text{if } z \in N(v) \\ 0 & \text{otherwise} \end{cases}$$

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There is a vector $\Phi \in \mathbb{R}^V_{>0}$ such that

$$\mathcal{C}\ \Phi \leq (1-\delta)\ \Phi$$

Connections with Loopy BP

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Jacobian of Loopy BP

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$J^* = J|_{\omega=\omega^*}$ denote the Jacobian of BP at the fixed point ω^*.

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Relation to Path Coupling

$$\mathcal{C} = D^{-1} J^* D,$$

where D is diagonal matrix, with $D(v, v) = \omega^*(v)$

Covergence from loopy BP

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Reduction to BP Spectral radius

There is a vector $\Phi \in \mathbb{R}^{V}_{>0}$ such that

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Spectral radius of BP in uniqueness region

We should expect $\rho(J^*) < 1$, because the fixed point ω^* is attractive

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Local Uniformity

Theorem

Let $\epsilon, \delta > 0$, $\Delta_0 = \Delta_0(\epsilon, \delta)$ and $C = C(\epsilon, \delta)$. Let G of max degree Δ, for $\Delta > \Delta_0$, and girth ≥ 7. For (X_t) the Glauber dynamics with fugacity $\lambda < (1-\delta)\lambda_c(\Delta)$ and any v the following holds: With probability $1 - \exp(-\Delta/C)$, we have that

$$\#\text{ Unblocked Neighbors of } v \text{ in } X_t < \sum_{z \in N(v)} \omega^*(z) + \epsilon\Delta$$

where $t \geq Cn \log \Delta$.

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Rapid Mixing with uniformity

Dyer, Frieze, Hayes, Vigoda 2013

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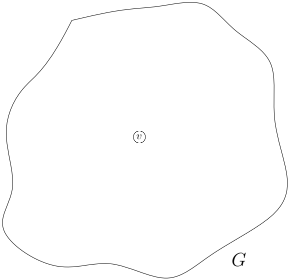

There is a single disagreement at v

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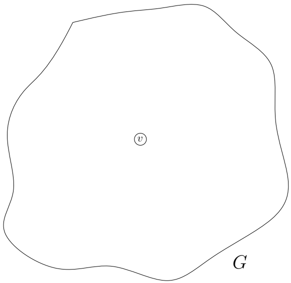

Run the chains for $Cn \log \Delta$ steps, “burn-in”

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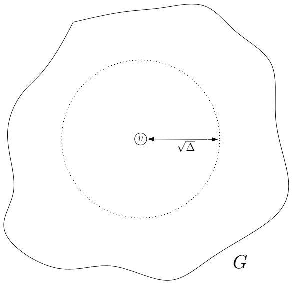

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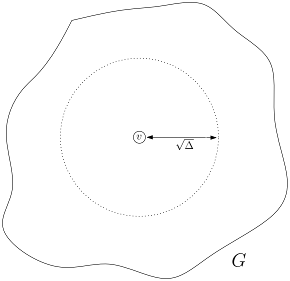

The disagreements spread in the graph during burn-in

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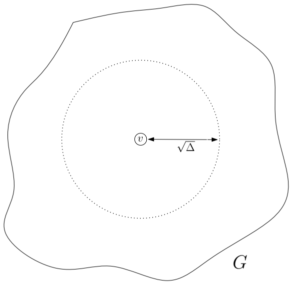

Typically the disagreements do not escape the ball

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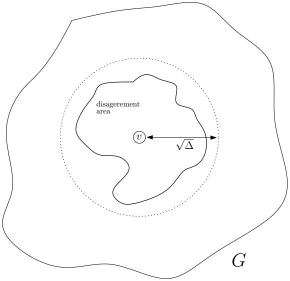

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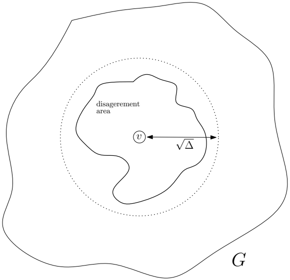

Typically the ball has uniformity.

Rapid Mixing with uniformity

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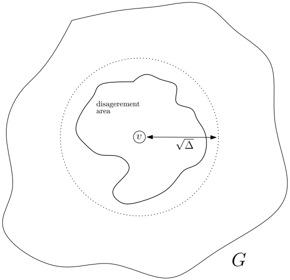

Interpolate and do path coupling for the pairs,
... pairs with have local uniformity

Rapid Mixing with uniformity

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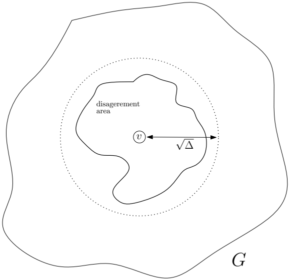
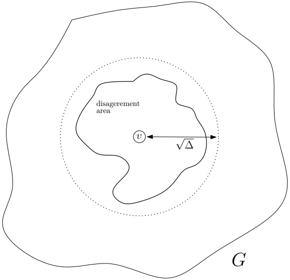


Interpolate and do path coupling for the pairs,
... pairs with have local uniformity and Φ gives contraction

Rapid Mixing with uniformity

Dyer, Frieze, Hayes, Vigoda 2013



$$\mathbb{E}\left[\Phi(X_{C'n\log\Delta}, Y_{C'n\log\Delta})\,\middle|\, X_0, Y_0\right] \leq (1-\gamma)\Phi(X_0, Y_0)$$

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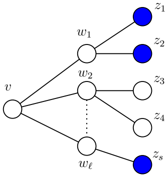

$$\mathbf{R}(\sigma, v) = \Pr_{Y \sim \mu}\left[v \text{ is unblocked in } Y | v \notin Y,\ Y(S_2(v)) = \sigma(S_2(v))\right]$$

Local uniformity I

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BP for Gibbs measure

Let $\gamma, \delta > 0$, $\Delta_0 = \Delta_0(\gamma, \delta)$ and $C = C(\gamma, \delta)$. Let G be of girth ≥ 6 and maximum degree $\Delta > \Delta_0$. Let X be distributed as in μ with $\lambda < (1-\delta)\lambda_c(\Delta)$.

Then for any vertex v with probability $\geq 1 - \exp(-\Delta/C)$ it holds that

$$\left| \mathbf{R}(X, v) - \prod_{z \sim v} \left(1 - \frac{\lambda}{1+\lambda} \mathbf{R}(X, z)\right) \right| < \gamma.$$

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BP for Glauber dynamics

Let G be of girth ≥ 7 and maximum degree $\Delta > \Delta_0$. Let (X_t) be the Glauber dynamics with $\lambda < (1-\delta)\lambda_c(\Delta)$.
Then for any vertex v and any $t > Cn \log \Delta$ with probability $\geq 1 - \exp(-\Delta/C)$ it holds that

$$\left| \mathbf{R}(X_t, v) - \prod_{z \sim v} \left(1 - \frac{\lambda}{1+\lambda} \mathbb{E}_{t_z}\left[\mathbf{R}(X_{t_z}, z)\right]\right) \right| < \gamma.$$

Local uniformity II

Local uniformity II

Lemma

Let G be of girth ≥ 7 and maximum degree $\Delta > \Delta_0$. Let (X_t) be the Glauber dynamics with $\lambda < (1-\delta)\lambda_c(\Delta)$.
For all $\mathcal{I} = [t_0, t_1]$, where $t_0 = Cn \log \Delta$, for every $v \in V$ with probability $1 - (1 + |\mathcal{I}|/n) \exp(-\Delta/C)$, we have that

$$(\forall t \in \mathcal{I}) \quad |\mathbf{R}(X_t, v) - \omega^*(v)| \leq \epsilon.$$

Iterations in space and time

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Convergence with Ψ

Potential function

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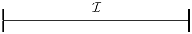

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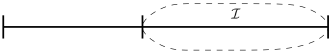

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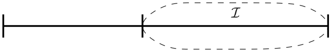

Provided

- $t \in \mathcal{I}'$ approximate BP equation hold in $B(v, R)$ $\forall t \in \mathcal{I}_{i+1}$, $u \in B(v, i+1)$

$$|\Psi(\mathbf{R}(X_t, u)) - \Psi(\omega^*(u))| \leq \alpha_{i+1}$$

$\forall t \in \mathcal{I}_i$, $u \in B(v, i)$

$$|\Psi(\mathbf{R}(X_t, u)) - \Psi(\omega^*(u))| \leq (1-\delta)\alpha_{i+1}$$

Iterations in space and time

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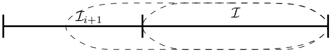

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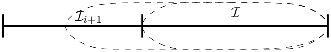

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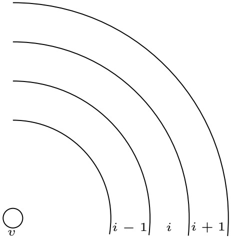

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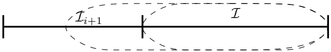

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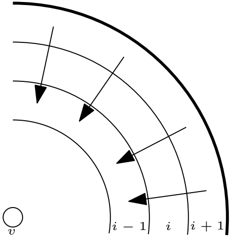

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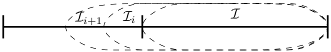

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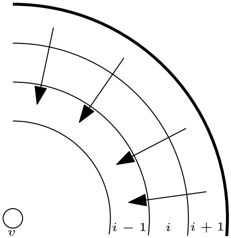

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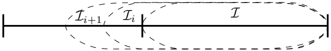

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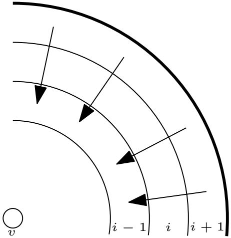

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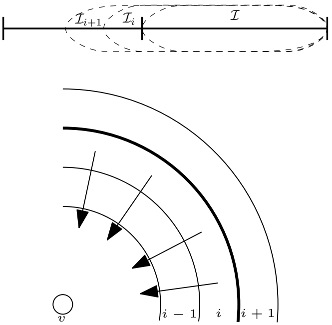

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THANK YOU!