Large Wireless Networks under Fading, Mobility, and Delay Constraints

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Abstract—We study wireless ad hoc networks with a large number of nodes, following the line of investigation initiated in [1] and continued in [2].

We first focus on a network of \( n \) immobile nodes, each with a destination node chosen in random. We develop a scheme under which, in the absence of fading, the network can provide each node with a traffic rate \( \lambda_1(n) = K_1(n \log n)^{-\frac{1}{2}} \). This result was first shown in [1] under a similar setting, however the proof presented here is shorter and uses only basic probability tools. We then proceed to show that, under a general model of fading, each node can send data to its destination with a rate \( \lambda_2(n) = K_2 n^{-\frac{1}{2}} (\log n)^{-\frac{1}{2}} \).

Next, we extend our formulation to study the effects of node mobility. We first develop a simple scheme under which each of the \( n \) mobile nodes can send data to a randomly chosen destination node with a rate \( \lambda_3(n) = K_3 n^{-\frac{1}{2}} (\log n)^{-\frac{1}{2}} \), and with a fixed upper bound on the packet delay \( d_{\text{max}} \) that does not depend on \( n \). We subsequently develop a scheme under which each of the nodes can send data to its destination with a rate \( \lambda_4(n) = K_4 n^{-\frac{1}{2}} (\log n)^{-\frac{1}{2}}, \) provided that nodes are willing to tolerate packet delays smaller than \( d_{\text{max}}(n) < K_5 n^d \), where \( 0 < d < 1 \). With both schemes, a general model of fading is assumed. In addition, nodes require no global topology or routing information, and only need to coordinate locally.

The above results hold for an appropriate choice of values for the constants \( K_i \), and with probability approaching 1 as the number of nodes \( n \) approaches infinity.

Keywords: System Design, Information Theory, Wireless Ad Hoc Network Capacity, Delay, Fading, Mobility.

I. INTRODUCTION

Wireless ad hoc networks consist of mobile nodes communicating over a wireless channel. Contrary to cellular networks, where the nodes are restricted to communicate with a few strategically placed base stations, in wireless ad hoc networks nodes are allowed to communicate directly. However, because of the nature of the wireless channel, each node can effectively communicate with only some of the others, typically those that lie in its vicinity. On the other hand, the traffic requirements are taken to be arbitrary, therefore it is necessary that nodes cooperate to forward packets to their final destinations.

The problem of determining fundamental limits on the performance of ad hoc networks has only recently attracted the interest of researchers [1], [2], [3], [4]. In a landmark paper, the authors of [1] investigate the asymptotic behavior of the capacity of a class of two-dimensional random networks as the number of nodes \( n \) approaches infinity, under a uniform traffic assumption. Here, nodes are assumed immobile. The authors present a scheme that achieves with high probability, i.e., with probability approaching 1 as \( n \) approaches infinity, a communication rate equal to \((n \log n)^{-\frac{1}{2}}\), up to a multiplicative constant, from each node to its randomly chosen destination. The authors also show that, with high probability, the \( n \) nodes cannot send data to their destinations with a per-node communication rate greater than \((n \log n)^{-\frac{1}{2}}\), up to a (different) multiplicative constant.

The last result is disheartening since it suggests that, as the number of nodes \( n \) goes to infinity, the per-node communication rate will necessarily go to zero. However, node mobility can lead to dramatic improvements: In [2] the authors concentrate on a wireless network with \( n \) mobile nodes. They show that, with high probability, in the absence of any constraint on the delay in the delivery of a packet, each node is guaranteed a fixed rate of communication to its destination, which is not a function of the number of nodes \( n \). The upside of this result is that, as the number of nodes increases, so will the expected delay experienced by the packets.

In this paper, we continue the investigation along the lines of [1] and [2]. In Section II we introduce our model for wireless ad hoc networks, and summarize our results. The next sections are devoted to the derivation of these results. In Section III we start by concentrating on networks with immobile nodes and with no fading. In Section IV we extend our formulation to include the effects of fading. In Section V we derive a per-node achievable rate of communication when nodes are mobile and there is a fixed bound on the delay, \( d_{\text{max}} \), which does not depend on the size of the network \( n \). In Section VI we derive an achievable per-node communication rate when nodes are mobile, but assuming that packet delays \( d_{\text{max}}(n) \) that increase with \( n \) are tolerated. We conclude in Section VII. Throughout
II. NETWORK MODEL AND OVERVIEW OF RESULTS

A. Immobile nodes with no fading

We consider a collection of \( n \) nodes \( X_1, X_2, \ldots, X_n \), placed randomly, uniformly and independently, in the two-dimensional area \( \{ (x, y) : -\frac{1}{2} \leq |x|, |y| \leq \frac{1}{2} \} \). For now, we take the nodes to be immobile.

Regarding the traffic model, we assume that each node is the source of a single stream, and the destination of a single stream. A node cannot be the source and destination of the same stream. Apart from this restriction, all other combinations of sources and destinations are equally probable. Alternative traffic models can easily be incorporated in our formulation and are studied in [5].

Nodes communicate over a wireless channel of bandwidth \( W \). Half-duplex transmission is assumed, i.e., nodes cannot transmit and receive simultaneously. Each node can transmit with any power \( P_i \), provided it is less than the maximum \( P_0 \). When node \( X_i \) transmits with power \( P_i \), node \( X_j \) receives the transmitted signal with power \( G_{ij} P_i \), where \( G_{ij} = K d_{ij}^{-\alpha} \). \( K \) is a constant, the same for all nodes, \( d_{ij} \) is the distance between nodes \( X_i \) and \( X_j \), and \( \alpha > 2 \) is the decay exponent. For now, we do not incorporate fading in our model.

Let \( \{ X_t : t \in T \} \) be the set of transmitting nodes at a given time, each node \( X_t \) transmitting with power \( P_t \). Let us assume that node \( X_t \), \( j \notin T \) is receiving information from \( X_i \), \( i \in T \). Then the signal to interference and noise ratio (SINR) at node \( X_j \) will be

\[
C_j = \frac{G_{ij} P_i}{\eta + \sum_{k \in T, k \neq i} G_{kj} P_k},
\]

where \( \eta \) is the thermal noise power at the receiver, which is assumed the same for all nodes. We assume that the transmission of the packet will be successful if and only if the transmission rate used, \( R_j \), satisfies the inequality

\[
R_j \leq f_R(C_j) \equiv W \log_2(1 + \frac{1}{\Gamma C_j})
\]

(1)

where \( \log_2(x) \) denotes the logarithm of \( x \), of base 2. With \( \Gamma = 1 \), the receiver achieves Shannon’s capacity. With \( \Gamma > 1 \), (1) approximates the maximum data rate that meets a given BER requirement under a specific modulation and coding scheme such as coded MQAM [6]. The precise value of \( \Gamma \) depends on the particular coding and modulation parameters. Other choices for the shape of \( f_R(\cdot) \) are also possible [5].

As in [1], each node is equipped with a buffer of infinite length, for storing packets intended for other nodes. We assume omniscient nodes with perfect knowledge of the topology. The transmission strategy for all nodes is agreed to in advance. Thus, no overhead is needed to determine the topology or the transmission strategy. Under the above set of assumptions, the following can be proved:

**Theorem 1:** (Immobile nodes with no fading) Each node can achieve, with high probability, a rate of communication equal to:

\[
\lambda_1(n) = \log_2(1 + \Gamma^{-1} \frac{3\alpha - 6}{3\alpha - 5} (10^{-\frac{\alpha}{2}})^{-1}) \left[ \frac{W}{72\sqrt{10}} \right] (n \log n)^{-\frac{1}{2}}.
\]

(2)

Note that a similar result was shown in [1], under a similar setting. Here, we present a proof that is shorter and more intuitive. In addition, the methodology used can be easily extended to handle fading and mobility, as shown with Theorems 2, 3, and 4.

B. Immobile nodes with fading

We now assume that when node \( X_i \) transmits with power \( P_i \), node \( X_j \) receives the transmitted signal with power \( G_{ij} P_i \), where \( G_{ij} = K d_{ij}^{-\alpha} \). The extra factor \( f_{ij} \) is the fading coefficient, a non-negative random variable that models fading, and does not change with time. We assume that the expectation \( E[f_{ij}] = 1 \), and that \( f_{ij} = f_{ji} \). We take the distinct \( n(n-1) \) fading coefficients to be independent and identically distributed (iid). We also assume that their complementary cumulative distribution function \( F^c(x) \) has a thin, exponentially decaying tail. Formally:

\[
F^c(x) \equiv P[f_{ij} > x] \leq \exp[-q x] \forall x > x_1,
\]

(3)

for some real and positive parameters \( q, x_1 \). In addition, we make the very mild assumption that there is a median value \( f_m > 0 \) such that \( P[f_{ij} \geq f_m] \geq 1/2 \). Both of these assumptions are satisfied by most distributions used to model fading. In particular, they are satisfied by the Nakagami, Ricean and Rayleigh distributions [5].

In this setting, we devise a scheme that guarantees a per-node communication rate that, compared to the previous case, is not reduced by more than a logarithmic factor. In particular, the following holds:

**Theorem 2:** (Immobile nodes with fading) Each node can achieve, with high probability, a rate of communication equal to:

\[
\lambda_2(n) = \left[ \frac{10^{-\frac{\alpha}{2}}}{648} \frac{3\alpha - 6}{3\alpha - 5} \Gamma \right] (n \log n)^{-\frac{1}{2}}.
\]

(4)

Theorems 1 and 2 would seem to imply that the traffic carrying capacity of the network is reduced in the presence of fading. This is not so, as the theorems only specify lower bounds on the capacity, in terms of uniformly achievable rates of communication. In fact we conjecture that, in our setting, fading increases the capacity of networks, but not by a factor that goes to infinity as \( n \) goes to infinity. If this is indeed the case, then our lower bound for the capacity in the presence of fading is as tight as our bound for the capacity in the non-fading case, up to a logarithmic factor. However, such an investigation is beyond the scope of this work.
C. Mobile nodes with fading and delay constraints

Let us assume that nodes are no longer immobile, but rather move inside the square region \( \{(x, y) : -\frac{1}{2} \leq |x|, |y| \leq \frac{1}{2}\} \), independently of each other, with the position of each node being uniformly distributed at any time, and according to a stationary and ergodic process that satisfies the following assumptions:

(i) There is a duration of time \( s \) such that within any interval of this duration all nodes remain immobile, so that the power gains of all links remain constant, even in the presence of fading.

(ii) There is a duration of time \( S \) such that after the passing of a time interval of length \( S \), the positions of all nodes become perfectly reshuffled. In other words, after the lapse of an interval equal to \( S \), the position of each node becomes randomly, uniformly and independently redistributed, irrespectively of the initial node placement.

The first assumption is a reasonable approximation of any realistic model of mobility, provided we take \( s \) to be sufficiently small. The second assumption holds for all stochastic mobility models that have finite memory. However it is also a reasonable approximation for traffic models in which the distribution of the position of a node that was at a particular location at time \( t = 0 \) approaches the uniform distribution as \( t \to \infty \). For convenience, we take \( S \) to be a large integer multiple of \( s \), i.e., \( S = Ns \).

Let the rest of the assumptions of Sections II-A and II-B continue to hold, with the exception that nodes no longer have global topology information. Also, the whole communication period will last for a time interval of length equal to \( 2n^2S \), where \( D \) is an arbitrary integer, greater than unity.

Before formally presenting the two main results, it is worthwhile discussing the basic idea behind them, which in fact is very simple: It was shown in [2] that a number of simultaneous transmissions on the order of \( n \) is possible, with a fixed transmission rate that does not decrease with \( n \), if all the nodes transmit to their nearest neighbors. In addition, because of the mobility of the nodes, two transmissions are enough for a packet: once to a relay (that happens to be the nearest neighbor of the source) and once more to the final destination, whenever this happens to be the nearest neighbor of the relay. (One transmission would also be enough, but the number of source-destination pairs that are also nearest neighbors is not on the order of \( n \).) Therefore, an aggregate throughput on the order of \( n \) is possible, so that the per-node throughput is fixed, and independent of the number of nodes \( n \). However, under this scheme each packet will have to remain in its relaying node for a time increasing proportionally with \( n \), since the chance of the relay being closest neighbor with the destination is \( \frac{1}{n} \), and that is excluding the queuing delay. (Of course, to make this argument formal we need to specify the mobility model in detail.)

The motivation for our schemes is simple: If more nodes could receive the packet when it is transmitted by its source (and act as potential relays), rather than just one, maybe the packet would not have to wait that much time. But for more nodes to receive the packet, it is necessary that fewer nodes transmit, so that the transmitted signals experience less interference and reach further. However, this will reduce the throughput. In other words, it is clear that, in principle, there can be a trade-off between delay and throughput. Indeed, we prove the following two theorems:

Theorem 3: (Mobile nodes with a fixed upper bound on packet delay) Under node mobility, fading, and an upper bound on the acceptable packet delay equal to \( d_{\max} = (N + 1)s \), each node can communicate with its destination with a rate equal to:

\[
\lambda_3(n) = \frac{W_f f_m}{72(D + 3)(10D + 20)\Gamma} \times \left[ \frac{3\alpha - 6}{3\alpha - 5} \right] n^{-\frac{5}{2}} (\log n)^{-\frac{1}{2}}. \tag{5}
\]

If larger delays are tolerated, the per-node communication rate can improve, as the following theorem shows:

Theorem 4: (Mobile nodes with a polynomial upper bound on delay) Under node mobility, fading, and an upper bound on the acceptable packet delay \( d_{\max} \leq (4Ns)\eta^D \), each node can communicate with its destination with a rate equal to:

\[
\lambda_4(n) = \frac{W_f m}{3456(D + 3)(11D)\Gamma} \times \left[ \frac{3\alpha - 6}{3\alpha - 5} \right] n^{\frac{1}{2}} (\log n)^{-\frac{1}{2}}. \tag{6}
\]

Theorem 4 suggests a fundamental tradeoff between the acceptable packet delay and the achievable per-node throughput: If we are willing to tolerate larger delays, our throughput will increase.

To better illustrate this trade-off, let us say that a particular communication scheme has a throughput exponent \( t \) if, with high probability, the per-node throughput is equal to \( \lambda(n) = Rn^t \), with \( R \) being a constant, modulo arbitrary powers of logarithms. Similarly, let us say that a communication scheme will have a delay exponent \( d \) if, with high probability, the maximum packet delay is bounded by \( d_{\max}(n) < Rn^d \), where \( R \) is a constant, modulo powers of logarithms.

Using this language, Theorem 4 implies that there are communication schemes whose throughput exponent and delay exponent pair lies on the open line interval \( AB \) of Fig. 1. Theorem 3 shows that point A is also achievable. Point B can be achieved by a scheme very similar to that of [2].

Although we take the parameters \( S, N \) and \( s \) to be constant, all our proofs would go through with no modification even if we had taken them to be functions of the number of users \( n \). In that case, however, the delay bounds of Theorems 3 and 4 would have a different dependence on \( n \).

III. IMMUNE NODES WITH NO FADING

In this section we prove Theorem 1. The proof is constructive, i.e., we will create a communications scheme, and show
that the scheme achieves the per-node communication rate of (2) with high probability.

A. The Cell Lattice

Let \([\lfloor x \rfloor]\) be the greatest odd multiple of 3 that is less than or equal to \(x\). Let \([\lfloor \frac{n}{2g(n)} \rfloor]\) = \(2r + 1\), where \(k_1\) is a constant to be specified later. We divide the square region \(\{(x, y) : -\frac{1}{2} \leq |x|, |y| \leq \frac{3}{2}\}\), where all the nodes are placed, in a regular lattice of \(g(n) = (2r + 1)^2\) cells. The boundaries of the cells are formed by the lines

\[ x = -\frac{1}{2} + \frac{i}{2r + 1}, \quad i = 0, \ldots, 2r + 1, \]
\[ y = -\frac{1}{2} + \frac{j}{2r + 1}, \quad j = 0, \ldots, 2r + 1. \]

We denote the cells by \(c_1, c_2, \ldots, c_{g(n)}\). In addition, each cell is identified by its coordinates \((v_1, v_2)\) in the lattice, where \(-r \leq v_1, v_2 \leq r\); the cell that contains the origin has coordinates \((0, 0)\). We define the coordinates of a node to be the coordinates of the cell in which the node lies (so two nodes may have the same coordinates). We call two cells neighbors if they share a common boundary edge, so that each cell has at most four neighbors. We call two nodes neighbors, if they lie in the same or neighboring cells. We will use the symbols \(\prec_a, \succ_a, \preceq_a, \succeq_a\) to denote that the corresponding inequality will only hold asymptotically, i.e., for sufficiently large \(n\). For example, \(f(n) \prec_a g(n)\) means that there is a \(n_0\) such that \(f(n) < g(n)\) for all \(n > n_0\). Using this notation, it is very simple to show that

\[ \frac{n}{2k_1 \log n} \prec_a g(n) \leq \frac{n}{k_1 \log n}. \]  

(7)

Our first step is to uniformly bound the number of nodes per cell. Let \(m_i\) be the number of nodes in cell \(c_i\). Then:

Lemma 1: (Number of nodes in cells)

\[ P\left[\frac{k_1 \log n}{2} \leq m_i \leq 4k_1 \log n \quad \forall i\right] >_a 1 - \frac{2n^{1-\frac{k_1}{2}}}{k_1 \log n}. \]

Proof: We make use of Chernoff’s bounds [7]: Let \(X\) be a binomially distributed random variable, with parameters \(k\) (the number of Bernoulli experiments) and \(p\) (the probability of success of each Bernoulli experiment). Then, for any \(\delta \in (0, 1]\),

\[ P[X < (1 - \delta)kp] < \exp(-kp \frac{\delta^2}{2}). \]  

(8)

Also, for any \(\delta > 0\),

\[ P[X > (1 + \delta)kp] < \exp[-kp f(\delta)], \]  

(9)

where \(f(\delta) = (1 + \delta)(1 + \delta) - \delta\). By calculating the derivative, we immediately have that \(f(\delta) > 0\) for \(\delta > 0\).

Let us concentrate on a particular cell \(c_i\). Then:

\[ P[m_i < \frac{k_1 \log n}{2}] \leq P[m_i < \frac{n}{2g(n)}] \quad \text{(Using (7))} \]
\[ < \exp[-\frac{n}{8g(n)}] \quad \text{(Using (8) with } \delta = \frac{1}{2}) \]
\[ \leq \exp[-\frac{k_1}{8} \log n] \quad \text{(Using (7))} \]
\[ = n^{-\frac{k_1}{8}}. \]  

(10)

In a similar manner, but using (9) instead of (8), it may be shown that

\[ P[m_i > 4k_1 \log n] <_a n^{-k_1 f(1)} < n^{-\frac{k_1}{8}}. \]  

(11)

We also note the inequality \(P(\cup_{i=1}^n E_i) \leq \sum_{i=1}^n P(E_i)\), typically referred to as the union bound. Combining (10) and (11) with the union bound, and using (7), we arrive at the result.

B. The routing rules

Lemma 1 shows that, with high probability, all cells are guaranteed to have at least one node (in fact many more), provided \(k_1\) is large enough. We are therefore justified to define the following rules that govern the routing of data to their final destination:

(i) Direct transmission of data is allowed only between nodes that lie in the same cell, or in neighboring cells (as was specified, cells are considered neighbors if they share a common edge, so that each cell has at most four neighbors).

(ii) Nodes that do not lie in the same or neighboring cells communicate by using nodes in intermediate cells as relays. The message is first transmitted along cells whose x-coordinate is the same as the x-coordinate of the source, until it arrives at a cell whose y-coordinate is the same as the y-coordinate of the destination. Then, the message is transmitted along cells whose y-coordinate is the same as the y-coordinate of the destination. In each of the intermediate cells, one of the nodes in the cell is arbitrarily selected to act as the relay of the packet.

Let \(n_i\) be the number of streams whose packets must be received (and possibly forwarded) by the nodes lying in cell \(c_i\). We have the following lemma:
Lemma 2: (Bound on the number of streams arriving at a cell)

\[
P[s_i \leq 8(k_1 n \log n)^{\frac{1}{2}} \forall i] > a - \frac{2n^{2-k_i}f(1)}{(k_1 \log n)^{\frac{1}{2}}}
\]

Proof: Because of the routing protocol, for each stream that passes through \(c_i\), at least one of the following two conditions must be satisfied: either the source and \(c_i\) share the same x-coordinate, or the destination and \(c_i\) share the same y-coordinate. It is also possible that both conditions are satisfied. Let \(s_{11}\) be the number of streams that satisfy the first condition, and \(s_{2}\) be the number of streams that satisfy the second condition. Clearly, \(s_i \leq s_{11} + s_{2}\).

Also, let \(M_i\) be the number of nodes that have the same x-coordinate with \(c_i\), and \(N_i\) the number of nodes that have the same y-coordinate with \(c_i\). We have

\[
P[M_i > 4(k_1 n \log n)^{\frac{1}{2}}] < a \frac{n^{\frac{1}{2}}-k_i f(1)}{(k_1 \log n)^{\frac{1}{2}}}, \quad (12)
\]

\[
P[N_i > 4(k_1 n \log n)^{\frac{1}{2}}] < a \frac{n^{\frac{1}{2}}-k_i f(1)}{(k_1 \log n)^{\frac{1}{2}}}. \quad (13)
\]

Indeed, \(M_i\) is Bernoulli distributed, so that:

\[
P[M_i > 4(k_1 n \log n)^{\frac{1}{2}}] \leq P[M_i > 4(k_1 n \log n(g(n))^\frac{1}{2})] \text{ (using (7))}
\]

\[
< a \frac{(g(n))^{\frac{1}{2}}-k_i f(1)}{(k_1 \log n)^{\frac{1}{2}}}, \text{ (union bound, symmetry, (11))}
\]

and we arrive at (12). The proof of (13) is similar.

By assumption, each node is the source of a single stream, so \(s_{11} \leq M_i\). Similarly, each node is the destination of a single stream, so \(s_{2} \leq N_i\). Combining these inequalities with the inequality \(s_i \leq s_{11} + s_{2}\), we have that \(s_i \leq M_i + N_i\). Applying the union bound and (12), (13), we have:

\[
P[s_i > 8(k_1 n \log n)^{\frac{1}{2}}] < a \frac{2n^{\frac{1}{2}}-k_i f(1)}{(k_1 \log n)^{\frac{1}{2}}}.
\]

By symmetry, this result holds for all cells \(c_i\). Therefore, applying the union bound once more we arrive at the result.

C. The time division scheme

By the construction of the cell lattice, if \(n\) is the number of nodes in the network, there will be \(g(n) = \left\lfloor \left( \frac{n}{k_1 \log n} \right)^{\frac{1}{2}} \right\rfloor^2 = (2r+1)^2\) cells, \(g(n)\) being a multiple of 9. Therefore, we may divide the \(g(n)\) cells perfectly into nine sub-lattices. We index the nine sub-lattices by the pairs \((i,j)\), where \(-1 \leq i, j \leq 1\). The cells belonging to sub-lattice \((i,j)\) are all those whose coordinates may be expressed as \((i + 3k_1, j + 3k_2)\), for some \(k_1, k_2 \in \mathbb{Z}\). In Fig. 2 we have shaded the cells belonging to one of the 9 sub-lattices.

We divide time into frames, and each frame into nine slots. Each slot corresponds to a sub-lattice. At any time during that slot, only one node from each cell of the sub-lattice is allowed to receive (but many nodes of that cell may receive consecutively in the same slot). As specified by the routing protocol, the transmitter of that transmission will have to lie in the same cell, or in one of the four neighboring cells. All transmissions will be with the maximum power \(P_0\).

Lemma 3: (Lower bound on the number of streams arriving at a cell)

In the absence of fading, the SINR \(C_j\) at node \(X_j\) is asymptotically lower bounded by

\[
C_j > a C_{\min}(n) \equiv \frac{3\alpha - 6}{3\alpha - 5} 10^{-\frac{2}{\alpha} - 1} \quad (14)
\]

for all \(j = 1, \ldots, n\) and with probability 1.

Proof: We first bound the interference. Let \(x_0 = \frac{1}{2r+1}\) be the length of the sides of the cells, and let \(c_j\) be the cell in which the receiver lies. Working as in [8], we note that the rest of the cells in the same sub-lattice are located along the perimeters of concentric squares, whose center is cell \(c_j\). For example, there are 8 cells along the perimeter of the first square (fewer if the cell is at the edge of the network). Irrespective of the coordinates of \(c_j\), all the cells of its sub-lattice are located along the perimeters of at most \(2r\) squares. There are at most \(8i\) interferers corresponding to the \(i\)-th square, whose distance from the receiver will be at least \(x_0(3i-2)\). Consequently, the interference at the receiver is upper bounded by

\[
I_j \leq \frac{\sum_{i=1}^{2r} \frac{8iK_0}{x_0(3i-2)}a}{\frac{8K_0}{x_0 a} \left[1 + \sum_{i=1}^{2r} \frac{i+1}{(3i+1)^\alpha} \right]} < \frac{8K_0}{x_0 a} \left[1 + \int_0^{2r} (3x+1)^{1-\alpha} dx \right] \leq \frac{8K_0}{x_0 a} \frac{3\alpha - 5}{3\alpha - 6} \left( \frac{n}{k_1 \log n} \right)^{\frac{2}{\alpha}}. \quad (15)\]

We also need a lower bound on the power of the useful signal. Clearly, since the maximum possible distance that the useful signal will need to travel, under the routing assumptions, is \(\sqrt{5}x_0\), the power of the useful signal \(S_j\) is bounded as follows:

\[
S_j \geq K_0(\sqrt{5}x_0)^{-\alpha} > a 10^{-\frac{2}{\alpha}} K_0 \left( \frac{n}{k_1 \log n} \right)^{\frac{2}{\alpha}}. \quad (16)
\]

Combining (15) with (16), and noting that the thermal noise remains bounded, and therefore becomes negligible as \(n \to \infty\), we arrive at (14). \(\square\)

D. Performance of the basic scheme

We will refer to the algorithm specified in Sections III-A through III-C as the basic scheme. We are now ready to prove our first main result, Theorem 1:

Proof of Theorem 1: We must show that, with high probability, the basic scheme achieves the per-node throughput given by (2). We set \(k_1 = 10 > \max(8, \frac{3}{\sqrt{10}})\). By Lemma 1, with high probability, i.e., with probability approaching 1 as \(n \to \infty\), each cell will contain a node that will be used
for routing. In addition, by Lemma 2 no cell will need to serve more than \(8(k_1 n \log n)^2\) routes, with high probability.

By Lemma 3, each receiver is guaranteed a reception rate of at least \(R_{min}(n) = f_R(C_{min}(n)) = W \log_2(1 + \Gamma^{-1} C_{min}(n))\), where \(C_{min}(n)\) is given by (14). Noting that, due to the time division scheme, each cell will be able to receive packets during only one out of nine slots, each stream is guaranteed, with probability approaching unity as \(n \to +\infty\), a rate equal to \(\lambda(n) = \frac{W \log_2(1 + \Gamma^{-1} C_{min}(n))}{9 \times 8(k_1 n \log n)^2}\). Substituting \(C_{min}(n)\) from (14) we arrive at the result. \(\square\)

### IV. Immobile Nodes with Fading

Under the basic scheme, each node with a packet waiting to be relayed through a neighboring cell will arbitrarily pick one of the nodes in that cell as the relay. In the absence of fading, the choice of relay is not critical. With fading, the choice is clearly important, as the power gains from one node to various nodes of the neighboring cell may be drastically different. Fortunately, by Lemma 1 there are many potential receivers for a transmitter to choose from, provided \(k_1\) is large enough.

We therefore use the basic scheme, with a value for \(k_1\) to be determined, and with a single modification on the routing rules. Specifically, when it is the turn of a node to transmit, the transmitter will pick as the receiver any of the nodes in the target cell, among those whose link with the transmitter has a fading coefficient greater than or equal to the median of the distribution, \(f_m\). (The probability that a node in the cell will satisfy this requirement is at least \(\frac{1}{2}\).) If such a node does not exist, the packet is dropped. This rule is extremely intuitive: nodes plainly avoid transmitting to nodes with poor fading coefficient. There is, however, a complication. Once a packet arrives to its destination cell, it may not find itself to its destination node. This can easily be amended as follows: Let the packet be in node \(X_1\), and have a destination node \(X_2\), both being in the same cell \(c_i\). The packet will be transmitted two more times, once to an intermediate node \(X_3\), and then from the intermediate node to the destination node \(X_2\). The intermediate node is chosen among the nodes in cell \(c_i\), such that the fading coefficients of both links are equal to or greater than \(f_m\). If no such node exists, the packet is dropped. We will refer to this algorithm as the spatial-diversity scheme.

For a given number of nodes \(n\), there is a positive probability that the packets of some of the streams will have to be dropped, because at some point along the way no node can be found to satisfy the restriction we place on the fading coefficients. However, as the number of nodes increases this probability goes to zero, if \(k_1\) is chosen large enough. To show this, let us first condition the discussion on the event \(E = \{k_1 \log n \leq m_i \leq 4k_1 \log n\} \forall i\).

The probability that a given node will not be able to send a packet to any of the nodes in a neighboring cell is at most \((\frac{1}{2})^\frac{\log 3}{\log 4} \log n\). There are \(n\) nodes, each with at most 4 neighboring cells. Using the union bound, we have that the probability that any of the nodes cannot forward a packet to one of its neighboring cells, because of fading, is upper bounded by \(4n(1 - \frac{\log 3}{\log 4})\).

Also, note that the probability that a given node will not be able to send a packet to another node in the same cell by means of a relay in that cell is at most \((\frac{1}{2})^\frac{\log 3}{\log 4} \log n\). There are \(n\) nodes, each with at most 4\(k_1\) log \(n\) potential destinations in the same cell. Using the union bound, we have that the probability that any one of the nodes cannot forward a packet to one of the other nodes in its own cell by means of a relay is upper bounded by \(4k_1 n \log n\).

By Lemma 1, the probability that the event \(E\) will not occur is asymptotically upper bounded by \(\frac{n \log n}{k_1 \log n}\). Let \(D\) be the probability that the packets of one or more of the streams are dropped. Then:

\[
P[D] \leq P[D|E] + P[E^c] \\
\leq 4n^{-\frac{\log 3}{\log 4}} + 4k_1 n^{-\frac{\log 3}{\log 4}} \log n \\
+ 2n^{-\frac{\log 3}{k_1 \log n}}.
\] (17)

Next, we need an upper bound on the value of the power gains \(f_{ij}\). Focusing on an arbitrary fading coefficient \(f_{ij}\), we have, using (3), that \(P[f_{ij}] > k_2 \log n\) \(\leq a \exp[-qk_2 \log n]\). Noting that there are \(\frac{n(n-1)}{2}\) fading coefficients, by use of the union bound we have:

\[
P[\max\{f_{ij}\} \leq k_2 \log n] \geq 1 - n^2 - qk_2.
\] (18)

We now are now ready to prove the main result of the section, Theorem 2:

**Proof of Theorem 2:** We show that, with high probability, the spatial-diversity scheme achieves the per-node throughput given by (4). Pick \(k_1 = 10 \max\{\frac{2}{\log 2}, \frac{2}{\log 4 - \log 3}, 8\}\). By (17), no packets will be dropped, with high probability.
Let us next concentrate on the SINR at the receivers. Because of the restriction we place on which paths may be used, if $S_j$ is the received power of the useful signal without fading and $S_j^F$ is the received power with fading, we will have $S_j^F \geq f_m S_j$. In addition, if we set $k_2 = \frac{3}{9}$ by (18), and with high probability, all fading coefficients will be smaller than $k_2 \log n$. Therefore, if $I_j$ is the interference power at $X_j$ with no fading and $I_j^F$ is the interference power at $X_j$ with fading, we will have that $I_j^F \leq (k_2 \log n)I_j$. This bound will hold uniformly, i.e. for all receptions, and with high probability. Noting that the thermal noise power is not affected with fading, we will have that

$$X = S - I = S - f_m S_j \leq \frac{1}{k_2 \log n} C_{\min}(n),$$

where $C_{\min}(n)$ is the minimum SINR in the absence of fading, given by (14). Therefore, with high probability, each receiver is guaranteed a rate $R_{\min}^F(n) = f_R(C_{\min}(n)) = W \log(2 + 2^{-1}C_{\min}(n))$.

By Lemma 2, a maximum of $8(k_1 n \log n)^{\frac{3}{2}}$ streams will be arriving at each cell. For a few of those, specifically those whose destination lies in that cell, three receptions will have to take place in each cell. Noting that each cell will be receiving in only one of every nine slots, we see that each stream is guaranteed, with high probability, a rate equal to $\frac{W \log(1+2^{-1}C_{\min}(n))}{216(k_1 n \log n)^{\frac{3}{2}}}$. Substituting $C_{\min}(n)$ from (19) and (14), and using the limit

$$\lim_{x \to 0} \log_2(1 + x) = \log_2 e > 1,$$

we arrive at the result. \qed

V. NODE MOBILITY WITH A FIXED DELAY CONSTRAINT

A. The fixed-delay scheme

As in Sections III and IV, we divide the area of the network in a regular lattice of $g(n)$ cells. Now, however, we require that $g(n) = \left\lfloor \left( \frac{n}{k_1 \log n} \right)^{\frac{1}{3}} \right\rfloor^2$, where $k_1$ will be specified later. The cells are again divided in the nine sub-lattices $(i, j)$ where $-1 \leq i, j \leq 1$. Note that

$$\frac{1}{2} \left( \frac{n}{k_1 \log n} \right)^{\frac{1}{2}} \leq g(n) \leq \left( \frac{n}{k_1 \log n} \right)^{\frac{1}{2}}.

As shown in Fig. 3(a), we divide time into frames, with each frame consisting of $N$ mini-frames. Each mini-frame will have a duration equal to $s$, and will consist of 9 slots, each of duration $\frac{s}{9}$. In each of the slots, only nodes lying in a corresponding sub-lattice are allowed to transmit (and only with maximum power). The rest will have to remain silent.

In addition to this time division, which is on the 9 slots of each mini-frame, we superimpose another, distinctly different time-division, which is on the $N$ mini-frames of each frame. Specifically, the $N$ mini-frames within the frame are logically independent: packets that are transmitted, received, created, etc., in a given mini-frame, are stored in the memory of the nodes for $N - 1$ mini-frames, and then brought back forward for the corresponding mini-frame of the next frame. We thus create $N$ independent virtual channels, with the nodes communicating using each of them independently. As a result, the topology in each of these virtual channels becomes totally decorrelated every other mini-frame, as opposed to every $N$ mini-frames.

In the following, we suppress all reference to the “outer” time division, except when we are doing delay calculations, and we concentrate on a single virtual channel. Therefore, our “working model” becomes that of Fig. 3(b).

The algorithm executed in each of the $N$ virtual channels is as follows: In the first, third, and generally all odd mini-frames, nodes wait for the slot in which they are allowed to transmit (this depends on which sub-lattice the node lies). They then transmit a packet, with rate $R_{\min}(n) = f_R(C_{\min}(n))$, where $C_{\min}(n)$ is given by (19), where $k_2$ will be specified shortly, for a time duration equal to $\frac{s}{9}$. Nodes that lie in the same cell coordinate among themselves, and transmit their packets consecutively. If there are more than $4(k_1 n \log n)^{\frac{1}{2}}$ nodes in any cell, then some of the packets will not be transmitted and will be dropped. We will call this a type-I error. The transmitted packets will possibly be received successfully by some of the other nodes lying in the same cell. These will act as relays in the subsequent mini-frame.

In the second, fourth, and generally all even-numbered mini-frames, each destination will wait for the slot in which nodes in its cell can transmit, and then will receive the packet intended for itself, by one of the relays lying in the same cell, that successfully received the packet in the previous mini-frame, and has a sufficiently strong path to the destination. We will say that a type-II error occurs if there are more than $4(k_1 n \log n)^{\frac{1}{2}}$ nodes in a cell, so that some of them will not have the time to receive their packets. A node is a potential relay if it is in...
the same cell with the source when the source is transmitting the packet, and it is in the same cell with the destination in the next mini-frame, and all fading coefficients are such that both transmissions will be successful. We will say that a type-III error occurs when, for a specific packet, no potential relay exists, so that the packet never makes it to the destination. At the end of each even-numbered mini-frame, nodes discard all packets remaining in their buffers. We will refer to this algorithm as the fixed-delay scheme.

B. No packets are dropped

We now show that the probability that an error of type-I, II, or III occurs at any time, at any part of the network, goes to zero as \( n \rightarrow \infty \), so that, with high probability, no packets are dropped.

We first bound the number of nodes at each cell of each mini-frame. Let \( m_{ij} \) be the number of nodes in cell \( c_i \), \( i = 1, \ldots, g(n) \) of mini-frame \( j \), \( j = 1, \ldots, 2Nn^D \). Then:

\[
P[m_{ij} < \frac{1}{2}(k_1n \log n)^{\frac{1}{2}}] \\
\leq P[m_{ij} < \frac{n}{2g(n)}] \quad \text{(Using (21))} \\
< \exp[-\frac{n}{8g(n)}] \quad \text{(Using (8) with } \delta = \frac{1}{2}) \\
\leq \exp[-\frac{1}{8}(k_1n \log n)^{\frac{1}{2}}] \quad \text{(Using (21))} \tag{22}
\]

Similarly, and using (9) instead of (8), we have that

\[
P[m_{ij} > 4(k_1n \log n)^{\frac{1}{2}}] < a \exp[-f(1)(k_1n \log n)^{\frac{1}{2}}]. \tag{23}
\]

Applying the union bound with (22) and (23), we arrive at:

\[
P\left[\sum_{i,j} \frac{1}{2}(k_1n \log n)^{\frac{1}{2}} \leq m_{ij} \leq 4(k_1n \log n)^{\frac{1}{2}} \forall i,j\right] \\
\rightarrow 1 \quad \forall k_1 > 0. \tag{24}
\]

By (24), type-I and type-II errors are excluded with high probability.

Next, we exclude type-III errors. For this, we first bound the value of the fading coefficients. Let \( f_{ijk} \) be the fading coefficient between nodes \( X_i \) and \( X_j \), at mini-frame \( k \), and let the event \( F = \{ \max\{f_{ijk}\} < k_2n \log n \} \). Noting that there are \( 2Nn^D \) mini-frames, by the union bound and (18) we have

\[
P[F] \geq a \rightarrow 1 - 2Nn^{D+2} - qk_2. \tag{25}
\]

We now condition the discussion on \( F \). The probability that a fading coefficient will equal or exceed \( f_m \) is not necessarily greater than \( \frac{1}{2} \), but is greater than \( \frac{1}{3} \) for sufficiently large \( n \). A condition that is sufficient (but not necessary) for a packet to be successfully received when the transmitter and receiver lie in the same cell with rate \( R_{min}(n) = f_n(C_{min}^F(n)) \), where \( C_{min}^F(n) \) is given by (19), is that the fading coefficient between the receiver and transmitter is equal to or greater than \( f_m \). This result follows from noting that the bounds (15) and (16) that were derived in the context of the basic scheme can be carried to the fixed-delay scheme with no modification. Indeed, in the fixed-delay scheme the transmitter of the useful signal must be placed closer (in the same cell as the receiver), and the interferers must be placed further than in the case of the basic scheme.

Excluding the source and the destination, there are \( n - 2 \) potential relays. The probability that one of them will be in the same cell as the source in the mini-frame when the packet leaves the source, and will be in the same slot as the destination in the subsequent mini-frame, and will have sufficiently strong paths to both source and destination, in both mini-frames, is \( p(n) \geq \frac{1}{g(n)} \frac{1}{g(n)} \frac{1}{2} \geq \frac{1}{4}\left(\frac{k_1 \log n}{n}\right) \). Therefore, the probability \( p_f \) that none of the \( n - 2 \) potential relays will be successful in relaying the packet is bounded by:

\[
p_f \leq (1 - \frac{k_1 \log n}{n})^{-2} < a \frac{n^{k_1}}{n}. \tag{26}
\]

The last inequality follows from basic properties of the exponential and the logarithmic functions. Since the total number of packets is \( N_n^{D+1} \), by the union bound, the probability that a type-III error occurs conditioned on the event \( F \), \( P[III|F] \leq a \left(\frac{n}{N_n^{D+1}}\right) + N_n^{D+2} - qk_2 \).

Removing the conditioning on \( F \), we have that the probability of a type-III error occurring, \( P[III] \), is bounded by:

\[
P[III] \leq P[III|F] + P[F^c] \\
\leq a \left(\frac{n}{N_n^{D+1}}\right) + N_n^{D+2} - qk_2. \tag{27}
\]

Therefore, by setting \( k_1 = 10(D + 2) \) and \( k_2 = \frac{D + 3}{4} \), type-III errors are excluded with high probability.

C. Performance of the fixed-delay scheme

Proof of Theorem 3: Regarding the packet delay, we assume that packets are created just before it is their node’s turn to transmit. (Delays due to random arrival times and queuing are beyond the scope of this work — such issues are discussed in [9].) We then note that each packet will arrive at its destination within an interval \((N + 1)s\) after its transmission from the source. Therefore, packet delays are smaller than a maximum \( d_{max} = (N + 1)s \). This bound does not depend on the number of nodes \( n \), but rather on the underlying mobility model: it is slightly longer than the decorrelation time of the topology.

To calculate the per-node throughput achieved by the scheme, we start by noting that exactly one packet of size \( \frac{c}{g(n)}(k_1n \log n)^{-\frac{1}{2}} R_{min}^F(n) \) bits will be arriving at each destination every 2 mini-frames, of duration 2s. Therefore, the per-stream throughput will be \( \lambda(n) = \frac{1}{g(n)}(k_1n \log n)^{-\frac{1}{2}} R_{min}^F(n) \). Noting that \( R_{min}(n) = W \log_2(1 + F^{-1} C_{min}^F(n)) \), substituting \( C_{min}^F(n) \) from (19), (14), and using the limit of (20), we arrive at the result. \( \square \)

The theorem states that a non-trivial aggregate throughput is possible when nodes are mobile, even with a fixed upper bound on the acceptable packet delay and without the nodes using global routing and topology information. This aggregate throughput increases like \( n^{\frac{k_1}{2}}(\log n)^{-\frac{1}{2}} \). This happens to be equal, up to a constant, to the aggregate throughput that nodes would roughly achieve if they were using the spatial-diversity
scheme, properly modified to handle node mobility, and assuming that they had access to global topology information. However, for large networks and high node mobilities, such information comes at a prohibitive cost, or is just impossible to acquire. But network designers need not despair: Theorem 3 states that the same aggregate throughput is achievable, with no need for routing or global topology information, provided nodes coordinate within their single-hop neighborhood, and a packet delay roughly equal to the topology decorrelation time is tolerated.

VI. NODE MOBILITY WITH A POLYNOMIALLY INCREASING DELAY CONSTRAINT

A. The polynomial-delay scheme

Let us assume that packet delays up to

$$d_{\text{max}}(n) = 2([n^d] + 1)Ns \leq (4Ns)n^d$$  \hspace{1cm} (27)

are acceptable, where $0 < d < 1$. We assume that packets are created just before their source’s turn to transmit. We employ the usual cell lattice, where the number of cells is now equal to

$$g(n) = \left\lfloor \frac{n^{d+1}}{k_1 \log n} \right\rfloor^2.$$  

The parameter $k_1$ will be determined later. Clearly,

$$\frac{1}{2} \left( \frac{n^{d+1}}{k_1 \log n} \right)^{\frac{1}{2}} < g(n) \leq \left( \frac{n^{d+1}}{k_1 \log n} \right)^{\frac{1}{2}}.  \hspace{1cm} (28)$$

Again, we divide time in frames, each frame in $N$ mini-frames, and each mini-frame in nine slots of duration $\frac{1}{9}$, as shown in Fig. 3(a). As previously, mini-frames are organized in $N$ virtual channels.

As with the fixed-delay scheme, within each of the $N$ virtual channels, odd numbered mini-frames are devoted to source-relay communication, and even-numbered mini-frames are devoted to relay-destination communication. During the odd-numbered slots, nodes behave similarly to the fixed-delay scheme; Each one waits for the turn of the slot that corresponds to its cell, and transmits a packet with rate $R_{\text{min}}(n) = f_R(C_{\text{min}}^{E}(n))$, where $C_{\text{min}}^{E}(n)$ is given by (19). The parameter $k_2$, appearing in (19), will be specified shortly. Each node will transmit for a duration of time equal to $\frac{9}{4}(Q(n)k_3 \log n)^{-1}$, where $k_3$ is a constant that will be specified later and we have used the shorthand

$$Q(n) = 4(k_1 n^{1-d} \log n)^{\frac{1}{2}}.$$ 

Nodes that lie in the same cell coordinate, and transmit their packets consecutively. If the number of nodes in a cell is greater than $Q(n)k_3 \log n$, then some of the packets will not be transmitted and will be dropped. We will call this a type-I error. The transmitted packets will be received by some of the other nodes lying in the same cell. These will act as relays in the subsequent mini-frames.

Contrary to the fixed-delay scheme, where each relay keeps packets for a single mini-frame, relays now keep the packets for the next $([n^d] + 1)$ even-numbered mini-frames. After that time, packets are removed from their buffers, as the delay constraint specified in (27) can no longer be satisfied.

In the second, fourth, and generally all even-numbered mini-frames, each destination will wait for the slot in which nodes in its cell can transmit, and then will attempt to receive all packets intended for it, by all relays that happen to be in the same cell. The relays will coordinate among themselves and transmit consecutively. Relays will only transmit if their fading coefficient to the destination is greater than $f_m$. In addition, if more than two relays have the same packet, then only one of them will transmit it. We will say that a type-II error occurs if there are more than $Q(n)k_3 \log n$ distinct packets in a cell, so that some of them may not have the time to be transmitted to their final destination. We will say that a type-III error occurs when, for a specific packet, there is no relay with sufficiently strong paths between both itself and the source, and itself and the destination, so that the packet never makes it to the destination, in any of the $([n^d] + 1)$ even-numbered mini-frames for which the packet waits in its relays. We will refer to this algorithm as the polynomial-delay scheme.

B. No packets are dropped

Here, we show that no packet is dropped, with high probability. Note that some packets may be received by their destination node multiple times, as relays do not coordinate among themselves and many of them might transmit the same packet to its destination, at different mini-frames. (This implies lost bandwidth, but the per-stream throughput is not reduced by more than a logarithmic factor.) Because of this, the system has natural redundancy, and the occurrence of a type-II error does not imply that a packet is lost. However, to prove that no packet is lost, it suffices to show that no errors of any type will be occurring.

Type-I errors are excluding by replicating the steps that led to (24) to show that:

$$P[\frac{Q(n)}{8} \leq m_{ij} \leq Q(n) \forall i, j] \rightarrow 1 \forall k_1 > 0. \hspace{1cm} (29)$$

We now exclude type-II errors. Let $P(\text{II})$ be the probability that a type-II error will occur at some mini-frame and cell. $p_{\text{II}}$ be the probability of a type-II error in a particular cell and cell-mini-frame, and $p_{\text{II}}$ be the same probability conditioned on the fact that there are $m$ nodes in that cell and mini-frame. We first focus on bounding $p_{\text{II}}$. Let the nodes be indexed by $X_1, \ldots, X_m$, and let $Y_1, \ldots, Y_m$ be the numbers of distinct packets intended for each of them, and carried by one or more of the rest of the nodes in the cell. Since each packet will require for its transmission a length of time equal to $\frac{9}{4}(Q(n)k_3 \log n)^{-1}$, we must ensure that there are no more than $Q(n)k_3 \log n$ of them. So we have:

$$p_{\text{II}}^m = P[\sum_{i=1}^{m} Y_i > Q(n)k_3 \log n] \
\leq P[\max_{i=1}^{m} Y_i > \frac{Q(n)k_3 \log n}{m}] \
\leq mP[Y_i > \frac{Q(n)k_3 \log n}{m}]. \hspace{1cm} (30)$$
Now there are at most \( \lfloor n^{d} \rfloor + 1 \) packets, indexed by \( P_1, \ldots, P_{\lfloor n^{d} \rfloor + 1} \) for node \( X_1 \) that could be among the \( Y_1 \) existing in the cell (if the slot is within one of the first \( \lfloor n^{d} \rfloor \) even-numbered mini-frames, the packets are fewer than \( \lfloor n^{d} \rfloor + 1 \) ). All of them were offered to their relays at distinct slots. Therefore, conditioned on the number of potential relays \( m \), the existence of any of them in the cell is independent of the existence of any other in the cell. If we remove the conditioning, the independence no longer holds: If a packet exists in the cell, this increases the probability that there are many nodes in the cell, and this in turn increases the probability that other packets will also exist in the cell. Therefore, conditioned on the number of nodes in the cell \( m \), \( Y_1 \) is binomially distributed, with at most \( \lfloor n^{d} \rfloor + 1 \) Bernoulli experiments and probability of success per experiment \( p_m \), where

\[
p_m \leq 1 - \left(1 - \frac{1}{g(n)}\right)^m \\
\leq a - \left(1 - \frac{2(k_1 \log n)^{\frac{3}{2}}}{n^{\frac{d+1}{2}}}\right)^m \quad \text{(using (28))} \\
\leq a - 1 - \exp\left[-3(k_1 \log n)^{\frac{1}{2}} n^{-\frac{d-1}{2}} m\right] \\
\leq a - 3(k_1 \log n)^{\frac{1}{2}} n^{-\frac{d-1}{2}} m.
\]

We can now bound \( p_{\text{II}} \). Let \( q_m \) be the probability that there are \( m \) nodes in the cell. We have:

\[
p_{\text{II}} = \sum_{m=1}^{[Q(n)]-1} P_{\text{II}}^m q_m + \sum_{m=[Q(n)]}^{n} P_{\text{II}}^m q_m \\
\leq \sum_{m=1}^{[Q(n)]-1} m P[Y_1 > \frac{Q(n)k_3 \log n}{m}] q_m \\
+ P[m > Q(n)] \quad \text{(using (30))} \\
\leq a - Q(n)P[Z > k_3 \log n] + P[m > Q(n)],
\]

where in the last inequality \( Z \) is a binomially distributed random variable, with \( \lfloor n^{d} \rfloor + 1 < 2n^{d} \) experiments, each with probability of success \( 12k_1 n^{-d} \log n \). Setting \( k_3 = 48k_1 \), and using (9), we have that \( P[Z > k_3 \log n] < \exp[-12f(1)k_1 \log n] \). Therefore,

\[
p_{\text{II}} \leq a - Q(n)n^{-12f(1)k_1} + P[m > Q(n)] \\
\leq a - 5(k_1 n^{-d} \log n)^{2} n^{-12f(1)k_1}.
\]

The last inequality comes by noting that, by (29), the first term dominates the second.

Applying the union bound, we have that the probability of a type-II error at any cell and even mini-frame is

\[
P(\text{II}) \leq a - 10Nn^{1-D-12f(1)k_1}.
\]

Setting \( k_1 > \frac{D+1}{12f(1)} \), we exclude type-II errors, with high probability.

Having excluded type-I and II errors, we know that no packet will be denied transmission because of an overcrowding of nodes. However, we still need to prove that all packets will be given the chance in the first place, i.e., we need to exclude type-III errors. There is, however, a technical issue that must be dealt with first: Those packets that will be created toward the end of the experiment, within a time \( d_{\text{max}}(n) \) from the end, will not stay in the relays as long as the rest. It is clear that some of them will never reach their destination. However, the fraction of these packets is smaller than \( \frac{(QNs)_{m}}{(QN)n^{m}} \). As this fraction goes to 0 as \( n \) increases, we will tolerate type-III errors for these packets.

We concentrate on a particular packet \( P \). Using the previous paragraph for justification, we assume that \( P \) is not created within a time period equal to \( d_{\text{max}}(n) \) from the end. We also condition the discussion on the event \( F = \{ \max\{f_{ijk}\} < k_2 \log n \} \), for which (25) continues to hold.

As in the fixed-delay scheme, conditioned on \( F \), a packet will be successfully received if the fading coefficient between the receiver and the transmitter is greater than or equal to \( f_m \) (and so with probability at least \( \frac{1}{4} \), for sufficiently large \( n \)). Counting the source and the destination, there are \( n \) potential relays. The probability that one of them in particular, say \( X_1 \), will receive the packet from the source is greater than or equal to \( \frac{1}{3 Q(n)} \). Assuming that this happens, the probability that \( X_1 \) will have the opportunity to forward the packet to the destination is at least \( 1 - (1 - \frac{1}{3 Q(n)})^{\lfloor n^{d} \rfloor + 1} \). So the probability that \( X_1 \) will be able to successfully act as a relay is \( p_1 = \frac{1}{3 Q(n)} \left[1 - (1 - \frac{1}{3 Q(n)})^{\lfloor n^{d} \rfloor + 1} \right] \). Noting that, for small \( x > 0 \), \( \log(1 + x) < x \) and \( 1 - e^{-x} \simeq x \), we have \( p_1 > a k_1 \log n \). However, there are \( n \) potential relays, so by a simple manipulation, we have that the probability that packet \( P \) will not be relayed by any node is upper bounded by \( n^{-\frac{k_3}{4}} \).

Using the union bound, the probability of a type-III error for any packet, conditioned on \( F \), \( P[\text{III} | F] \), is upper bounded by \( P[\text{III} | F] \leq a - Nn^{D-\frac{k_3}{4}} \).

Removing the conditioning on \( F \) as in (26), we have:

\[
P[\text{III}] \leq a - Nn^{D-\frac{k_3}{4}} + Nn^{D+2-qk_2},
\]

from which we have that by setting \( k_1 > 10D \) and \( k_2 > \frac{D+2}{q} \), type-III errors will not occur, with high probability.

### C. Performance of the polynomial-delay scheme

**Proof of Theorem 4:** We have set \( k_3 = 48k_1 \). We also set \( k_1 = 11D > \max\{10D, \frac{D+1}{12f(1)} \} \) and \( k_2 = \frac{D+3}{q} \). By Section VI-B, errors of all types are excluded with high probability.

We still need to calculate the throughput of the scheme. Each packet will have a size of \( R_{\text{min}}(n)\frac{Q(n)k_3 \log n}{n} \) bits. In each pair of mini-frames, duration \( 2s \), \( n \) such packets are created, all of them arriving at their destination (excluding a negligible fraction of those, all created in the last \( 4Nn^{D} \) mini-frames). This brings the per-stream communication rate to \( \lambda_4(n) = R_{\text{min}}(n)\frac{Q(n)k_3 \log n}{n} \). Noting that \( R_{\text{min}}(n) = W \log_q(1 + \Gamma^{-1} P_{\text{min}}^F(n)) \), substituting \( P_{\text{min}}^F(n) \) from (19), (14), and using the limit of (20), we arrive at the result.
VII. CONCLUSIONS

We derive lower bounds on the capacity of ad hoc wireless networks with a large number of nodes. Following [1] and [2], our approach is to construct schemes that achieve a given per-node throughput with probability approaching unity as the number of nodes increases.

We first study networks with immobile nodes, possibly in the presence of fading. We then study networks where the nodes are mobile, the channel exhibits fading, and delay constraints are placed on the delivery of packets. Our investigation leads to the establishment of a fundamental tradeoff between packet delay and throughput.

Because we do not resort to the law of large numbers or any other theorem of a similar flavor, we are able to identify in all cases the rate with which the probability that our schemes will work approaches unity (for example, see (17) and (18)).

It is assumed that the received power (or its expected value, in the case of fading) decays exponentially with distance, with an exponent $\alpha > 2$. Similar results can be derived when $1 < \alpha < 2$ [5]. Also, similar results will hold with more general traffic models [5].

Throughout this work it is assumed that all the fading coefficients are independent. A possible future line of research would be toward the establishment of bounds in the more general setting where fading coefficients are correlated.

REFERENCES


