Performance Bounds for Large Wireless Networks with Mobile Nodes and Multicast Traffic

S. Toumpis
Telecommunications Research Center Vienna (ftw.)
Donau-City-Straße, 1/3, A-1220, Austria, toumpis@ftw.at

A. J. Goldsmith
Department of Electrical Engineering
Stanford University, CA, 94305, andrea@ee.stanford.edu

Abstract— We investigate the traffic-carrying capabilities of wireless ad hoc networks with a large number of mobile nodes, under multicast traffic, packet delay constraints, and a general model for fading. We consider \( n \) nodes, each creating packets that must be delivered to around \( n^a \) (with \( 0 < a < 1 \)) of the rest of the nodes, chosen at random. We show that a simple time division scheme can achieve an aggregate throughput (measured at the destinations) on the order of \( n^{a-\epsilon} \), for any \( \epsilon > 0 \), and with a finite bound on the packet delay that does not increase with \( n \). Higher throughputs are also possible, but at the expense of packet delays that increase with \( n \). In particular, we present a scheme that achieves an aggregate throughput on the order of \( n^{1+\epsilon} \), for any \( \epsilon > 0 \), provided we tolerate packet delays on the order of \( nd \). With both schemes, nodes require no global topology or routing information, and only a minimal level of coordination. Our results hold with probability going to 1 as the number of nodes goes to infinity.

Keywords: Wireless Ad Hoc Network, Capacity, Delay, Fading, Mobility, Multicast Traffic, Throughput.

I. INTRODUCTION

The study of theoretical lower and upper bounds on the performance of wireless ad hoc networks with mobile nodes has recently attracted significant research interest [1], [2], [3], [4], [5], [6]. In [1], the authors showed that in networks with \( n \) mobile nodes, and in the absence of any constraint on the delay in the delivery of a packet, with probability going to 1 as \( n \) goes to infinity, each node can send data to a randomly chosen destination node with a fixed rate of communication, which is independent of \( n \). Therefore, the aggregate throughput can increase linearly with the number of nodes \( n \). In [2], the gains of having mobile relays supporting the communication of immobile nodes were explored. More recently, a number of researchers [3], [4], [6], [5] have independently investigated throughput-delay tradeoffs that exist in networks with mobile nodes.

Until now, to the best of the authors’ knowledge, studies in this field have assumed that each of the \( n \) nodes creates packets intended for a single destination, chosen randomly among the rest of the nodes. However, many envisioned applications of wireless ad hoc networks involve multicast traffic, in which a single packet has multiple recipients. For example, voice traffic between soldiers in the battlefield, communicating over a wireless network, would be a large extent multicast.

In this work, we develop performance bounds for a network with multicast traffic. The network consists of \( n \) mobile nodes, communicating over a wireless channel that exhibits flat fading. Each of the \( n \) nodes is creating packets that must be delivered to each of around \( n^a \) destinations (where \( 0 < a < 1 \)), chosen at random among the rest of the nodes. We first show a very simple time division scheme that can achieve an aggregate throughput (measured at the destinations) on the order of \( n^{a-\epsilon} \), for any \( \epsilon > 0 \), and with a maximum packet delay that does not increase with \( n \). We then proceed to show that higher aggregate throughputs are also possible, but at the expense of packet delays that increase with \( n \). In particular, an aggregate throughput on the order of \( n^{1+\epsilon} \) is possible, for all \( \epsilon > 0 \), provided we tolerate packet delays on the order of \( nd \). The design parameter \( d \) can take any value in the interval \((0,1)\). Therefore, the larger the value of \( d \) we select, the larger the throughput of our scheme, but also the greater the packet delay that we must tolerate.

The rest of the paper is organized as follows: in Section II we present our network model. In Section III we introduce some notation and a couple of technical lemmas that will be used repeatedly in the text. In Section IV we introduce the simple time division scheme, and in Section V we construct the more elaborate tradeoff scheme. We conclude in Section VI. Some proofs are sketched or omitted, but all proofs appear in [7].

II. NETWORK MODEL

Mobility model: We consider a collection of \( n \) nodes \( X_1, X_2, \ldots, X_n \), moving randomly, and independently of each other, in the two-dimensional area \( \{ (x, y) : |x|, |y| \leq \frac{1}{2} \} \). The position of any node at any particular time instant is uniformly distributed, and its movement can be described by a stationary and ergodic process that has the following properties:

(i) There is a duration of time \( s \), such that within this time interval, all nodes remain immobile, so that the power gains between all node pairs remain constant, even in the presence of fading.

(ii) There is a decorrelation time \( S \), such that after the passing of an interval of length \( S \), all memory is lost, and the positions of all nodes become perfectly reshuffled. In other words, after the lapse of an interval equal to \( S \), and conditioning on their initial positions, the nodes are again randomly, uniformly and independently redistributed. For convenience, we take \( S \) to be a large integer multiple of \( s \), i.e., \( S = Ns \).

Channel Model: Nodes communicate over a wireless channel of bandwidth \( W \). Half-duplex transmission is assumed, i.e., nodes cannot transmit and receive simultaneously. Each node can transmit with any power \( P_i \), provided it is less than a maximum \( P_0 \). When node \( X_i \) transmits with power \( P_i \), node \( X_j \) receives the transmitted signal with power \( G_{ij} P_i \), where \( G_{ij} = K d_{ij}^{-\alpha} f_{ij} \). \( K \) is a constant, same for all nodes, \( d_{ij} \) is the...
distance between nodes $X_i$ and $X_j$, and $\alpha > 2$ is the decay exponent. The factor $f_{ij}$ is the fading coefficient, a non-negative random variable that models fading; its value changes as nodes move. We assume that $E[f_{ij}] = 1$, and that $f_{ij} = f_{ji}$.

We take the distinct $n(n-1)/2$ fading coefficients to be independent and identically distributed (iid). We also assume that their complementary cumulative distribution function $F^c(x)$ has a thin, exponentially decaying tail. Formally:

$$F^c(x) \equiv P[f_{ij} > x] \leq \exp(-qx) \forall x > x_1,$$  \hspace{1cm} (1)

for some real and positive parameters $q$, $x_1$. In addition, we make the very mild assumption that there is a $f_m > 0$, the fading median, such that:

$$P[f_{ij} \geq f_m] \geq \frac{1}{2}.$$  \hspace{1cm} (2)

Both of these assumptions are satisfied by many distributions used to model fading. For example, it is straightforward to show that they are satisfied for the Nakagami, Ricean, and Rayleigh distributions, and for the trivial deterministic distribution, under which $f_{ij} = 1$ with probability 1.

**Receiver Model:** Let $\{X_t : t \in T\}$ be the set of transmitting nodes at a given time, each node $X_t$ transmitting with power $P_t$. Let us assume that node $X_i$, $j \not\in T$ is receiving information from $X_i$, $i \in T$. Then the signal to interference and noise ratio (SINR) at node $X_i$ will be

$$\gamma_{ij} = \frac{G_{ij}P_t}{\eta + \sum_{k \in T, \ k \neq i} G_{kj}P_k},$$

where $\eta$ is the thermal noise power at the receiver, which is taken to be the same for all nodes. We assume that the transmission of the packet will be successful if and only if the transmission rate used, $R_{ij}$, satisfies the inequality $R_{ij} \leq f_R(\gamma_{ij})$, where $f_R(\cdot)$ is a function that reflects the quality of the receiver and the performance metric. In particular, we use the formula:

$$f_R(\gamma_{ij}) = W \log_2(1 + \frac{1}{\Gamma} \gamma_{ij})$$  \hspace{1cm} (3)

where $\log_2(x)$ denotes the logarithm of $x$ of base 2. If $\Gamma = 1$, the receiver achieves Shannon’s bound. For $\Gamma \geq 1$, (3) approximates the maximum data rate that meets a given BER requirement for the given level of SINR and under a specific modulation scheme such as MQAM [8].

**Traffic Model:** Each node creates data packets with a rate $\lambda(n)$ bps, same for all nodes. The packets must be received by $m(n)$ distinct nodes, chosen at random among the rest of nodes. Each set of distinct $m(n)$ nodes has the same chance of being selected as any other set of $m(n)$ nodes. Each node makes its choice of $m(n)$ destinations independently of the rest.

We use the symbols $<, >$, $\leq, \geq$ to denote that the corresponding inequality only holds asymptotically, i.e., for sufficiently large $n$. Using this notation, we make the additional assumption that $k_1n^a < a m(n) < a k_2n^a$ for some $k_1, k_2 > 0$, and $a \in (0, 1)$.

We define the aggregate throughput $T(n)$ as $T(n) = m(n)\lambda(n)$. In other words, we measure the aggregate throughput at the packet destinations, and not at the packet sources.

Finally, we assume that the whole experiment lasts for a time interval of length equal to $2nD^N s$, where $D$ is an arbitrary integer, greater than unity.

**III. NOTATION AND TECHNICAL LEMMAS**

Unless specified otherwise, all limits we write will be for $n \to \infty$. We will say that $f(n)$ approaches a fixed limit $L$ exponentially fast with rate $r$ if $|f(n) - L| \leq a \exp(-kn^r)$, for some $k > 0$. We will then write $f(n) \to L$. Following [9], we will say that a sequence of events $\{A_n\}$ will occur with high probability (w.h.p.), if $P[A_n] \to 1$. The following lemma shows that the intersection of polynomially many events will occur w.h.p., provided the probability of these events goes to 1 exponentially fast:

**Lemma 1:** Let $\{A_{nm}\}$, where $n = 1, \ldots, m \leq M(n)$, be a collection of events for which $P[A_{nm}] = P[A_{n1}]$ for all $m = 1, \ldots, M(n)$, and $P[A_{n1}] \to 1$. Also let $M(n) \leq n^p$. Then $P[\bigcap_{m=1}^{M(n)} A_{nm}] \to r 1$.

The proof is a straightforward application of the union bound:

$$P[\bigcup_{n=1}^{\infty} E_i] \leq \sum_{i=1}^{\infty} P(E_i).$$

Then

$$P[\bigcup_{n=1}^{\infty} E_i] \leq \frac{\epsilon}{1 + \epsilon^{1+\delta}}$$

when $0 < \delta \leq 1$, and

$$P[\bigcup_{n=1}^{\infty} E_i] \leq \frac{\epsilon}{1 + \epsilon^{1+\delta}}$$

for all $\delta > 0$. The result is a straightforward application of (4) and (5).

**IV. THE TIME DIVISION SCHEME**

We now outline the very simple time division scheme, that can achieve an aggregate throughput in the order of $n^{\alpha-\epsilon}$, for any $\epsilon > 0$, with a finite packet delay that is not a function of the number of nodes $n$.

We allow only a single transmission at any given time, and with a rate equal to $W \log_2(1 + \frac{K \cdot X}{2 \pi^2 \sigma^2})$.

By our receiver model, if the fading coefficient between a node and the transmitter is constantly equal or greater than the median, then the node will receive the transmission, irrespective of the precise locations of the two in the area $\{x, y : |x|, |y| \leq \frac{1}{2}\}$.

We divide time into consecutive frames of duration $s$. By our mobility model, nodes do not move within the interval of a single frame, and fading coefficients remain constant. We also divide frames into $n$ slots, one for each node. We assume that an ordering among the nodes exists, so that the $i$-th slot of each frame is devoted for the transmission of a single packet created at the $i$-th node. Furthermore, each slot is divided into $r(n)$ minislots of equal duration, where $k_3n^\alpha < r(n) < k_4n^\alpha$, for some arbitrary $k_3, k_4, \epsilon > 0$.

We now outline the operation of the scheme for the duration of a slot that corresponds to, say, node $X$. During the first minislot, $X$ transmits a packet. With probability at most $\left(\frac{1}{2}\right)^{n(n-1)}$, none of the other $n - 1$ nodes receives the packet, because they all have a very small fading coefficient with $X$. In this event, no node will transmit anything for the rest
of the slot. Otherwise, there is at least one node, say $Y$, that receives the packet successfully. In this event, during the second minislot $Y$ retransmits the same packet again, and with probability at least $1 - (\frac{1}{2})^{n-2}$ a third node, say $Z$, other than $X$ and $Y$, receives it successfully. In the third minislot, $Z$ transmits the same packet, and so on. By the end of the slot, the same packet will have been transmitted at most $r(n)$ times, each time by a different node.

**Theorem 1:** Under the time division scheme, with high probability, i.e., with probability going to 1 as the number of nodes goes to infinity, all created packets are received by all of their destinations. In addition, this scheme can achieve an aggregate throughput (measured at the destinations) greater than $n^{a-\epsilon}$, for any $\epsilon > 0$, and all packets are delivered within a maximum delay of $d_{max}^{TD} = s$ seconds.

**Proof:** The probability $p(n)$ that a particular packet will be transmitted exactly $r(n)$ times is:

$$p(n) \geq \prod_{i=1}^{r(n)} [1 - (\frac{1}{2})^{n-i}] \geq_a [1 - (\frac{1}{2})^{\frac{n}{2}}]^{n^2}$$

$$\simeq \exp[n^{2\epsilon}(1 - (\frac{1}{2})^{\frac{n}{2}})] \simeq 1 - (\frac{1}{2})^{\frac{n}{2}} \ln 2 \to \frac{1}{2}.$$

Since $n$ packets are created in each frame, and there are polynomially many frames, by Lemma 1, the event $A$ that all packets will be transmitted exactly $r(n)$ times, by $r(n)$ distinct nodes, occurs w.h.p.

Conditioned on $A$, the probability that one of the $m(n)$ intended destinations of a packet will not receive the packet (because its fading coefficients with each of the $r(n)$ transmitters of the packet was too small) is at most $(\frac{1}{2})^{r(n)} \leq \epsilon \exp(-K ln 2)n!$. Since there are only polynomially many packets, each with only polynomially many destinations, by Lemma 1, each destination will receive the packet intended for it.

Since each node gains access to the channel once every $s$ seconds, the accumulated data at any node will not have to be buffered for more than a duration of $d_{max}^{TD} = s$. In addition, within each frame of duration $s$, $n$ packets of size $(nr(n))^{-1} W \log_2(1 + \frac{K}{n^2 + 1})$ are created. Each of them is delivered to $m(n)$ destinations, therefore the aggregate throughput, as measured at the destinations, is $\frac{m(n) W}{r(n)} \log_2(1 + \frac{K}{n^2 + 1}) \geq a n^{a-\epsilon}$. Since $\epsilon$ was chosen arbitrarily, we can make the substitution $2\epsilon \to \epsilon$, and he have that the aggregate throughput can exceed $n^{a-\epsilon}$, for any $\epsilon > 0$.

**V. THROUGHPUT-DELAY TRADEOFF**

In Section IV we showed that, if each packet must be delivered to around $n^n$ destinations, our time division scheme can achieve an aggregate throughput (measured at the destinations) on the order of $n^{a-\epsilon}$, for any $\epsilon > 0$, even in the presence of fading. Given the simplicity of that scheme, the natural question becomes whether we can do better. Here we introduce the **tradeoff scheme**, which outperforms the time division scheme in terms of throughput, but at the expense of packet delays than increase with the number of nodes.

**A. The Cell Lattice**

Let us assume that packet delays up to

$$d_{max}(n) = 2([n^d] + 1)Ns < (4Ns)n^d$$

are acceptable, where $0 < d < 1$. We denote by $|x|$ the greatest integer that is less than, or equal to $x$. Let $b$ be such that

$$1 < 2b = d + 1 - \epsilon,$$

where $\epsilon \in (0, d]$ but can be arbitrarily small. Let $[n^{\frac{1}{d}}] = r$. We divide the area occupied by the nodes, $\{(x, y) : |x|, |y| \leq \frac{n^{\frac{1}{d}}}{}\}$, into a **regular lattice** of $g(n) = r^2$ cells, as shown in Fig. 1. It is straightforward to show that

$$\forall \epsilon_1 > 0, (1 - \epsilon_1)n^{b} < a g(n) \leq n^{b}.$$

We arrange the cells in four **regular sub-lattices**, as shown in Fig. 1, such that any two cells in the same sub-lattice are separated by cells belonging to other sub-lattices.

**B. Frame Format**

We divide time into $2n^D$ identical frames, with each frame consisting of $N$ mini-frames. Each mini-frame has a duration equal to $s$, and consists of 4 slots, each of duration $\frac{s}{4}$. In each of the 4 slots, only nodes lying in one of the 4 corresponding sub-lattices are allowed to transmit (and only with maximum power). The rest will have to remain silent. In addition, at any time, only one node can be transmitting per cell, and only nodes in the same cell will be attempting to receive the transmitted data.
In addition to this “inner” time division, which is on the 4 slots of each mini-frame, we superimpose another, distinctly different, “outer” time division, which is on the $N$ mini-frames of each frame. Specifically, the $N$ mini-frames within the frame are logically independent: packets that are transmitted, received, created, etc., in a given mini-frame are stored in the memory of the nodes for $N-1$ mini-frames, and then brought back forward for the corresponding mini-frame of the next frame. We thus create $N$ independent virtual channels, with the nodes communicating using each of them independently. Successive mini-frames in each of these channels are separated by the decorrelation time of the node positions $S = Ns$. As a result, the topology in each of these virtual channels becomes totally decorrelated every other mini-frame, as opposed to every $N$ mini-frames.

C. A Few More Lemmas

Note that, according to the mobility model, nodes do not move for the duration of a mini-frame. Therefore, it makes sense to define $m_{ij}$ as the number of nodes in cell $c_i$ during mini-frame $j$. (Note that $i = 1, \ldots, g(n)$ and $j = 1, \ldots, 2Nn^D$.) Since $b < 1$, the number of cells is polynomially less than the number of nodes. This has the very nice implication that the nodes become uniformly distributed in the cells exponentially fast:

**Lemma 3:** (Number of nodes in cells) For all $\beta > 1$, 
\[
\Pr\left[ \frac{n^{1-b}}{b} < m_{ij} < \beta n^{1-b} \forall i,j \right] \to (1-b) 1.
\]

The proof is a simple application of Lemmas 1 and 2, and so is omitted, but can be found in [7].

Since nodes do not move for the duration of a mini-frame, the fading coefficients between any two of them remain constant. Therefore, it makes sense to define $f_{ijk}$ as the fading coefficient between nodes $i$ and $j$, where $1 \leq i < j \leq n$, during the $k$-th mini-frame, where $1 \leq k \leq 2Nn^D$. We then have:

**Lemma 4:** (Bound on the fading coefficients) For all $\epsilon_1 > 0$, \[
P[f_{ijk} < n^{\epsilon_1} \forall i,j,k] \to 1.
\]

**Proof:** We fix $i$, $j$, and $k$ and note that, by (1), 
\[
P[f_{ijk} < n^{\epsilon_1}] > 1 - \exp[-\gamma_n^{\epsilon_1}] \to 1.
\]
Since the number of fading coefficients increases polynomially fast, the result follows by applying Lemma 1.

We will also need a lower bound on the Signal to Interference and Noise Ratio (SINR) at each receiver. Intuitively, such a bound exists, as the time division scheme spaces out the interferers. In fact, working as in [2], we can prove the following (a detailed proof can be found in [7]):

**Lemma 5:** (Lower bound on the SINR) In the absence of fading (i.e., $f_{ijk} = 1$), the SINR $\gamma_i$ at node $X_i$, where $i = 1, \ldots, n$, is asymptotically lower bounded by a constant value $\gamma_{min}$, that is not a function of $n$: $\gamma_i \geq a \gamma_{min}$.

D. The Tradeoff Scheme

In the following, we suppress all reference to the “outer” time division, unless we are doing delay calculations, and we concentrate on a single virtual channel. It should be understood, though, that the nodes will be executing the same algorithm independently, and concurrently, in each of the $N$ virtual channels.

Odd numbered mini-frames are set aside for source-relay communication, and even-numbered mini-frames are dedicated to relay-destination communication. During the odd-numbered mini-frames, each node waits for the turn of the slot that corresponds to its sub-lattice, and transmits a new packet with rate $R_{min}(n) = fR_F(\gamma_{min}(n))$, where $\gamma_{min}(n)$ is given by

\[
\gamma_{min}(n) = (f_mn^{-m})\gamma_{min},
\]

where $\gamma_{min}$ was determined in Lemma 5, and $\epsilon$ is defined in (7). Therefore, the node transmits assuming an SINR at the destination which is smaller by a factor of $(f_mn^{-m})$ from the SINR at the receiver if there had been no fading. This factor is used to cushion the effects of fading. The node will transmit for a duration of time equal to $s/[4n^{1-b+a+2\epsilon}]^{-1}$. Nodes that lie in the same cell coordinate, and transmit their packets consecutively. If there are more than $n^{1-b+a+2\epsilon}$ nodes in a cell, then some of the packets will not be transmitted and will be dropped, however by Lemma 3 this will not occur w.h.p.

The transmitted packets will be received by some of the other nodes lying in the same cell. These will act as relays in subsequent even-numbered mini-frames. The relays keep the packets in their buffers for the next $(\lfloor n^\delta \rfloor + 1)$ even-numbered mini-frames. After that time, packets are removed from their buffers, as the delay constraint (6) can no longer be satisfied.

In the second, fourth, and generally all even-numbered mini-frames, each destination will wait for the slot in which nodes in its cell can transmit, and then will attempt to receive all packets intended for it, by all relays that happen to be in the same cell. The relays will coordinate themselves and transmit consecutively. Relays will only transmit if their fading coefficient to the destination is greater than $f_m$. In addition, if more than two relays have the same packet, then only one of them will transmit it. We will say that a type-A error occurred if there are more than $n^{1-b+a+2\epsilon}$ distinct packets in a cell during some mini-frame, so that some of them may not have the time to be transmitted to their final destination. We will say that a type-B error occurs when, for a specific packet, there is no relay with sufficiently strong paths between both itself and the source, and itself and the destination, so that the packet never makes it to the destination, in any of the $(\lfloor n^\delta \rfloor + 1)$ even-numbered mini-frames it remains in the system.

Note that there is a strictly positive probability that at a given cell and mini-frame there is a relay node carrying a packet, and also two or more of the around $n^a$ destinations for that packet. According to our scheme the relay may attempt to transmit the same packet two or more times, once for every destination. However, we are interested in achieving an aggregate throughput $n^t$ with $t > a$. (Indeed the simple time division scheme of the previous section achieves an aggregate throughput on the order of $n^a$.) In that case, as we will show shortly, we need to have $b > a$. Therefore, there will be polynomially more cells than destinations per packet, and it is a simple application of Lemmas 1 and 2 to show that, w. h. p., there will never be two destinations of the same packet in the same cell, at the same mini-frame.

E. Exclusion of Errors

Note that some packets may be received by the destination node multiple times, as relays do not coordinate among themselves and many of them might transmit the same packet to its destination, at different slots. (This implies lost bandwidth, but the exponent of the aggregate throughput is not reduced by more than $\epsilon$.) Because of this, the system has natural redundancy, and the occurrence of a type-A error does not imply that a packet is lost. However, to prove that no packet is lost, it suffices to show that the probability of errors of type-A and type-B goes to 0 as $n \to \infty$. 

References
We exclude type-B errors: We first note that those packets that are created toward the end of the experiment, within a time \( d_{\text{max}}(n) \) from the end, do not stay in the relays as long as the rest. It is clear that some of them will never reach their destination. However, the fraction of these packets is smaller than \( \frac{(4N)n^d}{(2N)n^d} \). As this fraction goes to 0 as \( n \) increases, we tolerate type-B errors for these packets.

Since there are polynomially many packets, we concentrate on a particular one \( P \). Using the previous paragraph for justification, we assume that \( P \) is not created within a time period equal to \( d_{\text{max}}(n) \) from the end. We also condition the discussion on the event \( E \) that no fading coefficient will be larger than \( n' \) at any slot. By Lemma 4, \( P[E] \rightarrow 1 \). Let \( P_B \) be the probability of a type-B error on this packet, and \( P_B^E \) the same probability conditioned on \( E \).

Conditioned on \( E \), the interference experienced at a receiver will not be greater than \( n' \) times the interference the receiver would experience if there was no fading. Therefore, by (9), if a transmitting and a receiving node lie in the same cell, the packet will be successfully received if the fading coefficient between them is greater than or equal to \( 1/n' \), and so with probability greater than \( 1/4 \), for sufficiently large \( n' \). Excluding the source and the destination, there are \( n - 2 \) relays. The probability that one of them in particular, say \( X_1 \), will receive the packet from the source is greater than \( 1/\binom{n}{3} \). Assuming that this happens, the probability that \( X_1 \) will have the opportunity to forward the packet to the destination in one of the following \( \lfloor (n^d) + 1 \rfloor \) even mini-frames is greater than \( p(n) = [1 - (1 - \frac{1}{\binom{n}{3} n^d})^{\lfloor n^d + 1 \rfloor} \geq 1/4 \). Noting that, for small \( x \), \( \log(1 + x) \approx x \) and \( 1 - e^{-x} \approx x \), we can lower bound \( p(n) \) by \( p(n) \geq a_k n^d - b_k \), for some \( k \geq 0 \). The number of nodes that will be able to act as relays is binomially distributed, with \( a(n) = n - 2 \) and \( p(n) \geq a_k n^d - b_k \). Noting that we took \( 2b = d + 1 - \epsilon \), we have \( a(n)p(n) \geq a_k n^d - b_k \), for some \( k \geq 0 \). So Lemma 2 applies and the probability that at least one node will be able to act as relay goes to 1 exponentially fast, with rate \( \epsilon \). Since the total number of packets increases polynomially with \( n \), the probability \( P_B^E \) of a type-B error at any time, conditioned in \( E \), goes to zero exponentially fast. We can now remove the conditioning on \( E \), to bound the unconditional probability \( P_B \) that a type-B error occurs at any time during the experiment:

\[
P_B \leq P_B^E + (1 - P[E]) \rightarrow 0.
\]

The exclusion of type-A follows along very similar lines, and so is omitted.

**F. Throughput Calculation**

We start by noting that each packet will have a size of \( R_{\text{min}}(n) \). In each pair of mini-frames, of duration \( 2\xi \), \( n \) such packets are created, all of them arriving at their \( m(n) \) destination w. h. p. (excluding a negligible fraction of those, all created in the last \( \lfloor n^d + 1 \rfloor \) frames). This brings the aggregate throughput to \( T(n) = m(n)R_{\text{min}}(n)2^{k_{n^d - d}} = m(n)\left(\sum_{n'}^{n} \log_2(1 + \gamma_{\text{min}}(n))\right) \). Bypartitioning \( \gamma_{\text{min}}(n) \) from (9), noting that \( \lim_{x \to 0} \log_2(1 + x) = \log_2(e) \), and using (7), we arrive at \( T(n) \geq a_k n^d - b_k - 2\epsilon \). Setting \( 4\epsilon \rightarrow 0 \), we have:

\[
\text{Theorem 2: The mobile nodes can achieve an aggregate throughput (measured at the destinations) } T(n) = \frac{n^{d - 2\epsilon}}{a_k}, \text{ for any } \epsilon > 0, \text{ while packet delays remain bounded by } d_{\text{max}}(n) = 2N s[(n^d + 1) < (4N) n^d], \text{ with probability going to 1 as } n \to \infty, \text{ and for any choice of } d \in (0, 1)\).

**VI. Conclusions and Future Work**

We study ad hoc wireless networks under multicast traffic conditions and node mobility, and under a general model for fading. We show that an aggregate throughput on the order of \( n^{d - 2\epsilon} \) can be achieved by a simple time division scheme, which in addition ensures that all packets are received within a finite delay, that does not increase with \( n \). If the nodes are willing to endure packet delays on the order of \( n^{d - 2\epsilon} \), then aggregate throughputs on the order of \( n^{d - 2\epsilon} \), for any \( \epsilon > 0 \), are also achievable.

These results hold with high probability, i.e., with probabilities approaching 1 as the number of nodes goes to infinity. Furthermore, the convergence is exponentially fast. This suggests that our results can also provide insight even in the design of networks with a modest number of nodes.

We concentrate only on throughputs of the form \( kn^d \). Therefore, we do not attempt to include any factors that are slower than polynomial, such as logarithmic factors. We choose to introduce this approach for two reasons: Firstly, our proofs become simpler and shorter. Secondly, there may be settings in which an aggregate throughput of the form \( kn^d \), for any \( \epsilon \in (-\infty, 0) \), is possible, but not for example an aggregate throughput of the form \( n^d f(\log n) \), where \( f(\cdot) \) is a rational function. Therefore, the scope of this approach may be wider.

For example, it was assumed in this work that nodes within the same cell are allowed to schedule consecutive transmissions with no communication overhead. We conjecture that the need for local coordination can be substituted with a random access scheme, for example Aloha, with a reduction in throughput not exceeding a factor of \( n' \), for any \( \epsilon > 0 \). This is the subject of future work.

**References**


