Power Allocation over Parallel Gaussian Multiple Access and Broadcast Channels

Gautam A. Gupta, Student Member, IEEE, and Stavros Toumpis, Member, IEEE

Abstract—We determine the optimal power allocation, that achieves any specified point on the boundary of the capacity region, for sets of parallel Gaussian Multiple Access Channels (MACs) and sets of parallel Gaussian Broadcast Channels (BCs). The power allocation is across the parallel channels and, within each channel, across the users. In both cases, there is a single constraint on the total power used. First, the allocation for the parallel MACs is determined, in a simple form, using the Karush-Kuhn-Tucker (KKT) conditions and a simple Lagrangian argument. Using this result, the allocation for the parallel BCs is derived using recent findings on the duality of the MAC and BC.

Index Terms—Broadcast Channel, Capacity Region, Duality, Multiple Access Channel, Parallel Channels, Power Allocation.

I. INTRODUCTION

The Gaussian Multiple Access Channel (MAC) and the Gaussian Broadcast Channel (BC) are the most common modeling choices whenever there is many-to-one or one-to-many communication over a common channel, notably in the cases of the cellular uplink and downlink. On the other hand, sets of parallel channels appear in the modeling and analysis of various channels, for example channels with Intersymbol Interference (ISI), fading, Time/Frequency/Code Division Multiple Access, and so on. Consequently, parallel Gaussian MACs and BCs have attracted significant research interest.

A first study of parallel Gaussian BCs appears in [1]. There, an algorithm is given for calculating the optimal allocation of power among the different parallel channels and among the users of each channel. The power allocation is optimal in the sense that it can achieve any specified point on the boundary of the capacity region. In [2], [3] an alternative greedy algorithm is offered.

In [4], Gaussian BCs with ISI and colored noise are modeled as sets of parallel BCs, each BC corresponding to a different frequency. The authors study the optimal power allocations across different users and frequencies, using the results of [1]. In [5], Gaussian fading BCs are modeled as sets of parallel channels, each channel corresponding to a different fading state. The authors compare the optimal power allocation across different users and states, determined by the method of [1], with a number of suboptimal, but much simpler, power allocations.

Parallel MACs are studied in [6], in the context of MACs with ISI. Similarly to [4], the authors show that such channels can be modeled as sets of parallel MACs, each corresponding to a different frequency. The authors determine graphically the optimal power allocation across different frequencies and users. Parallel MACs are also studied in [7], in the context of channels with fading. In this work, and similarly to [5], each parallel channel corresponds to a different fading state.

In this work, we calculate optimal power allocations, that achieve any point on the boundary of the capacity region, for parallel Gaussian MACs and BCs. Contrary to previous works, [6], [7], we assume a sum-power constraint for the MAC, meaning that there are no constraints on the powers of individual transmitters, but rather a global constraint on the total power across all users and channels.

The sum-power constraint is worth investigating firstly because it appears naturally in various applications. For example, consider a wireless sensor network that consists of a number of sensors $T_m$, each relaying data with rate $R_j$ to a central site. If the total power that can be allocated to this network, for example in the form of batteries, is fixed, but we are free to distribute it as we like, we would like to know the optimal distribution of batteries. As another example, consider the uplink of a cellular network, in which a constraint has been placed on the sum of transmitter powers, so as to bound the amount of interference caused at nearby cells. Secondly, recent duality results [8] have shown that the capacity region of the MAC with a sum-power constraint is equal to the capacity region of an appropriately defined dual BC, and in addition the power allocation that achieves a given point in the capacity region of the BC can be calculated using the power allocation that achieves the same point in the MAC. We use this duality to derive the power allocations of the parallel BC from the power allocations of the parallel MAC, in a very straightforward manner.

The rest of this work is organized as follows: We start in Section II by calculating the optimal power allocation in the case of a single Gaussian MAC, under a sum-power constraint. The proof is based on the application of KKT conditions. Our methodology is similar to the approach used in [1] for finding the power allocation of broadcast channels. However, our proving strategy (for example, we establish early on that the problem is convex) leads to shorter derivations and simpler, explicit expressions for the optimal power allocation and corresponding points on the capacity regions. In Section III we use an implicit Lagrangian argument to extend our result to the case of sets of parallel MACs. In Section IV we calculate the optimal power allocations for a single Gaussian BC and for sets of parallel Gaussian BCs, using the results of Sections II and III and duality arguments [8]. We conclude in Section V.

II. THE $m$-USER MULTIPLE ACCESS CHANNEL

A. The MAC with Individual Power Constraints

The $m$-user Gaussian Multiple Access Channel (MAC) consists of $m$ transmitters $T_1, T_2, \ldots, T_m$, communicating with a single receiver $V$. At integer time instances $i$ each transmitter $T_j$ sends a real-valued signal $X_j[i]$ to the receiver. The receiver receives the composite signal

$$Y[i] = \sum_{j=1}^{m} \sqrt{h_j} X_j[i] + n[i],$$

where $h_j > 0$ is the constant power gain between $T_j$ and $V$, and $n[i]$ is Gaussian noise with zero mean and variance $\sigma^2$. The noises $n[i]$ at different time indices $i$ are statistically independent. Each transmitter $T_j$ can transmit with any power at time index $i$, as long as the long-term time average of the transmitted power is less than $P_j$. We use the notation $h = (h_1, h_2, \ldots, h_m)$ and $P = (P_1, P_2, \ldots, P_m)$.

The capacity region $C_{MAC}(P, h)$ of this channel, i.e., the set of all rate vectors $R = (R_1, R_2, \ldots, R_m)$ such that $T_1, T_2, \ldots, T_m$ can simultaneously send data to the receiver with rates $R_1, R_2, \ldots, R_m$, respectively, is known to be the following polyhedron [9]:

$$C_{MAC}(P, h) = \left\{ R \geq 0 : \sum_{j \in S} R_j \leq \frac{1}{2} \log \left( 1 + \frac{1}{\sigma^2} \sum_{j \in S} h_j P_j \right) \right\},$$

for each $S \subset \{1, \ldots, m\}$. It can be shown that this polyhedron has exactly $m!$ vertices in the positive quadrant (i.e., with all their elements positive), each
corresponding to exactly one of the \( m! \) distinct permutations of the set \( \{1, 2, \ldots, m\} \). The vertex \( R_\pi = (R_{\pi(1)}, R_{\pi(2)}, \ldots, R_{\pi(m)}) \), which corresponds to permutation \( \pi(\cdot) \), is given by

\[
R_{\pi(l)} = \frac{1}{2} \log \left( 1 + \frac{h_{\pi(l)} T_{\pi(l)}}{\sigma^2 + \sum_{i=\pi(l)+1}^{m} h_{\pi(i)} T_{\pi(i)}} \right), \quad l = 1, \ldots, m. \tag{2}
\]

To achieve this combination of rates, the receiver successively decodes the signals coming from the different transmitters in the order specified by \( \pi(\cdot) \), i.e., the signal of \( T_{\pi(1)} \) is decoded first, the signal of \( T_{\pi(2)} \) is decoded second, etc. As suggested by (2), during the decoding of the signal from transmitter \( T_{\pi(l)} \), the signals already decoded (i.e., those coming from transmitters \( T_{\pi(1)} \) to \( T_{\pi(l-1)} \)) do not interfere with the decoding. However, those signals not yet decoded (i.e., those coming from transmitters \( T_{\pi(l+1)} \) to \( T_{\pi(m)} \)) will appear as additional noise.

The following lemma has appeared in a number of different contexts [7], [10], [11], and is a direct result of the polymatroid structure of the capacity region [7].

**Lemma 1:** Consider the following optimization problem:

\[
\text{maximize: } \mu \cdot R, \quad \text{subject to: } R \in \mathcal{C}_{\text{MAC}}(P; h),
\]

where the priority vector \( \mu = (\mu_1, \mu_2, \ldots, \mu_m) \geq 0 \). The maximum is achieved at exactly the convex hull of those vertices \( R_\pi \) of the capacity region that correspond to permutations \( \pi \) that place the priorities in non-decreasing order:

\[
h_{\pi(1)} \leq h_{\pi(2)} \leq \ldots \leq h_{\pi(m)}. \tag{3}
\]

**B. Problem Formulation**

We now assume that the transmitters do not have individual constraints on their power, but rather there is a single sum-power constraint \( P \cdot 1 \leq P_0 \). The capacity region of the resulting channel is the union of all capacity regions over all possible allocations of power to the individual transmitters that satisfy the sum-power constraint [8]:

\[
\mathcal{C}_{\text{MAC}}^S(P_0; h) = \bigcup_{\forall \pi: T_{\pi} \neq \emptyset} \mathcal{C}_{\text{MAC}}(P; h). \tag{4}
\]

Note that \( \mathcal{C}_{\text{MAC}}^S(P_0; h) \) is convex. Indeed, let \( R_1 \) and \( R_2 \) belong to \( \mathcal{C}_{\text{MAC}}^S(P_0; h) \). Therefore, \( R_1 \in \mathcal{C}_{\text{MAC}}(P_1; h) \) and \( R_2 \in \mathcal{C}_{\text{MAC}}(P_2; h) \) for some \( P_1, P_2 \). By a time division argument, for any \( \alpha \in (0, 1) \), the rate vector \( \alpha R_1 + (1-\alpha) R_2 \) will belong to \( \mathcal{C}_{\text{MAC}}(\alpha P_1 + (1-\alpha) P_2; h) \), and so will also belong to \( \mathcal{C}_{\text{MAC}}^S(P_0; h) \).

It follows by the convexity of \( \mathcal{C}_{\text{MAC}}^S(P_0; h) \), that its boundary can be completely traced out by solving the following maximization problem [12]:

\[
\text{maximize: } \mu \cdot R \quad \text{subject to: } R \in \mathcal{C}_{\text{MAC}}^S(P_0; h), \tag{5}
\]

for all priority vectors \( \mu = (\mu_1, \mu_2, \ldots, \mu_m) \) such that \( \mu \geq 0 \) componentwise. In more detail, consider the point \( R \in \mathcal{C}_{\text{MAC}}^S(P_0; h) \) that maximizes the weighted sum \( \mu \cdot R \), for some \( \mu \geq 0 \). Provably, this point will lie on the boundary of \( \mathcal{C}_{\text{MAC}}^S(P_0; h) \). In addition, all points on that boundary appear as a solution of (5) for some \( \mu \geq 0 \). In this section, we calculate in closed form the rate vector and the associated power allocation that solves (5).

Note also that, in (1), and hence also in (5), the channel gains \( h_j \) and powers \( P_j \) appear only through the products \( h_j P_j \). Therefore, more general constraints on the powers of the form \( \mathbf{P} \cdot \mathbf{a} \leq P_0 \) can be brought in the simpler form \( \mathbf{P} \cdot \mathbf{1} \leq P_0 \) by considering the rescaled channel for which \( \mathbf{h}' = \mathbf{h}/\mathbf{a} \) componentwise.

**C. Constraints on the Priorities and Channels Gains**

In order to simplify the problem (5), we first prove the following lemma:

**Lemma 2:** For the purposes of solving (5), we can assume, with no loss of generality, that:

\[
0 < h_1 \leq h_2 \leq \ldots \leq h_m, \tag{6}
\]

\[
h_1 > h_2 > \ldots > h_m > 0. \tag{7}
\]

**Proof:** To prove (6), we first note that, with no loss of generality, we can order the transmitters in terms of increasing priorities: \( h_1 \leq h_2 \leq \ldots \leq h_m \). Now let us assume that the first \( j \) priorities, \( 0 < j < m \), are zero: \( h_1 = h_2 = \ldots = h_j = 0 \). It is intuitively clear that under the optimal power allocation, the first \( j \) transmitters should not be allocated any power. Indeed, assume that some of them will get some positive power. Lemma 1 applies, and the maximum will be achieved by successive decoding in an order of non-decreasing priorities. If we remove the power from the first \( j \) transmitters, and give it to the first transmitter to be decoded after them, then the objective will strictly increase, which is a contradiction. Therefore, transmitters with 0 priority get no power, and we can drop them from consideration and relabel the rest with no loss of generality. To conclude:

\[
0 < h_1 \leq h_2 \leq \ldots \leq h_m. \tag{8}
\]

This inequality can be further sharpened to give (6). For this, let us consider two transmitters \( T_i \) and \( T_j \) that share the same priority, that is, \( h_i = h_j \), and chosen so that the signals from \( T_i \) and \( T_j \) are decoded consecutively. Let us assume that the powers allocated to them are \( P_i \) and \( P_j \), respectively, and let \( I \) be the combined power of the noise and those signals that will be decoded after them. If the signal from \( T_i \) is decoded directly before the signal from \( T_j \), we have:

\[
R_i + R_j = \frac{1}{2} \log \left( 1 + \frac{h_i P_i}{I + h_j P_j} \right) + \frac{1}{2} \log \left( 1 + \frac{h_j P_j}{I} \right) = \frac{1}{2} \log \left( 1 + \frac{h_i P_i + h_j P_j}{I} \right).
\]

Clearly, if \( h_i = h_j \), the objective function will not change if some of the power allocated to one of them is given to the other. On the other hand, if the channel gains \( h_i \) and \( h_j \) are not equal, then the transmitter with the smaller gain should not be given any power. It follows that, among all transmitters with the same priority (which may be more than two), only those with the maximum gain can be allocated a positive power, and the power of the rest will be zero. Among those transmitters who share the maximum gain, all allocations of power between them will make the same contribution to the objective function. Therefore, with no loss of generality, we can assume that one of them, chosen arbitrarily, will get positive power.
and the rest will get no power at all, and so we can remove them from consideration and relabel the remaining transmitters. Therefore, for each priority level there will be only one transmitter, and we can sharpen (8) to derive (6).

As the priorities are now distinct, by Lemma 1 it follows that there is only one decoding order which achieved the maximum, the one that corresponds to the identity permutation \( \pi(l) = l \).

Next, we derive (7). For this, let us consider two consecutive transmitters \( T_i \) and \( T_{i+1} \), for which \( h_i \leq h_{i+1} \), and let us assume that, under the optimal power allocation, \( P_i > 0 \). Also, let \( I \) be the combined power of the noise and the signals from transmitters \( T_{i+2}, \ldots, T_m \). The contribution of \( T_i \) and \( T_{i+1} \) to the objective function will be:

\[
\frac{1}{2} \mu_i R_i + \mu_{i+1} R_{i+1} = \frac{1}{2} \mu_i \log(1 + \frac{h_i P_i}{I + h_{i+1} P_{i+1}}) + \frac{1}{2} \mu_{i+1} \log(1 + \frac{h_{i+1} P_{i+1}}{I})
\]

\[
< \frac{1}{2} \mu_{i+1} \log(1 + \frac{h_i P_i}{I} + h_{i+1} P_{i+1})
\]

\[
\leq \frac{1}{2} \mu_{i+1} \log(1 + \frac{h_{i+1}(P_i + P_{i+1})}{I}).
\]

Therefore, we can do better by taking the power of \( T_i \) and giving it to \( T_{i+1} \), thus arriving at a contradiction about the optimality of the allocation. It follows that \( P_i = 0 \). To conclude, we cannot have \( \mu_i < \mu_{i+1} \), \( h_i \leq h_{i+1} \) and \( P_i > 0 \). By using induction, it is straightforward to show that this rule also extends to pairs of transmitters that are not consecutive: we cannot have \( \mu_i < \mu_j \), \( h_i \leq h_j \) and \( P_i > 0 \), for any \( j > i \). This is very intuitive: no transmitter should be given any power if there is another transmitter with strictly higher priority and the same or better channel. For the rest of this work, we disregard those transmitters, and relabel the rest as \( T_1, T_2, \ldots, T_m \). Therefore, with no loss of generality we can assume (7). \( \square \)

\[D. \text{ Convexity}\]

Using (2), and noting that the optimal permutation \( \pi() \) is the identity permutation \( \pi(l) = l \), we can write the optimization problem (5) as follows:

\[
\begin{align*}
\text{maximize:} & \quad \frac{1}{2} \sum_{l=1}^{m} \mu_l \log(1 + \frac{h_l P_l}{\sigma^2 + h_h P_h}), \\
\text{subject to:} & \quad \sum_{k=1}^{m} P_k \leq P_0, \quad P_i \geq 0, \quad i = 1, \ldots, m,
\end{align*}
\]

where the priorities \( \mu_l \) and channel gains \( h_i \) satisfy (6) and (7) respectively. This optimization problem can be written as:

\[
\begin{align*}
\text{minimize:} & \quad f_0(P_1, \ldots, P_m) = \frac{1}{2} \sum_{l=1}^{m} \mu_l \log(\sigma^2 + \sum_{k=1}^{m} h_k P_k), \\
\text{subject to:} & \quad \sum_{k=1}^{m} P_k \leq P_0, \quad P_i \geq 0, \quad i = 1, \ldots, m,
\end{align*}
\]

where we set \( \mu_0 = 0 \).

We note that the function \( f_0 \) is strictly convex. Indeed, it can be written as the composition

\[
f_0(P_1, \ldots, P_m) = h(a_1(P_1, \ldots, P_m), \ldots, a_m(P_1, \ldots, P_m)),
\]

where \( h : [\sigma^2, \infty)^m \to \mathbb{R} \) is defined by \( h(a_1, \ldots, a_m) \triangleq \frac{1}{2} \sum_{l=1}^{m} \mu_l \log(\sigma^2 + \sum_{k=1}^{m} h_k P_k) \), and the functions \( a_i : [0, \infty)^m \to [0, \infty) \) are defined by

\[
a_i(P_1, \ldots, P_m) \triangleq \sigma^2 + \sum_{k=1}^{m} h_k P_k, \quad i = 1, \ldots, m.
\]

The Hessian of \( h \) is positive definite everywhere in its domain, so \( h \) is strictly convex. Also, the mapping \((P_1, \ldots, P_m) \to (a_1, \ldots, a_m)\) is affine. Therefore, by a straightforward application of the definition of strict convexity [13], it follows that the composition (10) is strictly convex.

As the constraints of (9) are all linear, and the objective function \( f_0() \) is convex, it follows that the optimization problem (9) is convex. Also, note that the domain of the problem, i.e., the set where the constraints are satisfied, is compact, and \( f_0() \) is continuous. Therefore, the infimum is achieved and it makes sense to talk of a minimum. Moreover, from the strict convexity of \( f_0() \) it follows that the minimum is achieved for a single power allocation.

Finally, the optimization problem is clearly feasible (i.e., all the constraints are satisfied in a non-empty set), and all the constraints are linear. Therefore, Slater’s condition is satisfied [13], so that the following Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for a power allocation to be optimal:

\[
\sum_{k=1}^{m} P_k = P_0, \quad P_i \geq 0, \quad \frac{\partial f_0}{\partial P_i} + \nu \geq 0, \quad P_i \left[ \frac{\partial f_0}{\partial P_i} + \nu \right] = 0, \quad (12)
\]

for \( i = 1, \ldots, m \), and for some \( \nu \in \mathbb{R} \).

Setting \( \lambda = 2\nu \log 2 \) and computing the partial derivatives, the conditions (12) become:

\[\sum_{k=1}^{m} P_k = P_0, \quad (13)\]

\[h_i \sum_{l=1}^{i} \frac{\mu_l - \mu_{l-1}}{a_l} - \lambda \leq 0, \quad (15)\]

\[P_i \left[ h_i \sum_{l=1}^{i} \frac{\mu_l - \mu_{l-1}}{a_l} \right] = 0, \quad (16)\]

for \( i = 1, \ldots, m \) and some \( \lambda \in \mathbb{R} \), and with \( a_l \) as defined in (11).

\[E. \text{ The optimal power allocation if the active set is known}\]

We now derive the optimal power allocation, assuming for the moment that the active set, i.e., the set of those transmitters that are allocated positive power, is known. Note that a very similar approach was taken in [14], where the maximum transport capacity of the BC was determined. There, however, a particular assumption was adopted on the relation between the priorities and channel gains, that significantly simplified the computations, but limited the scope of the results. So let us assume that exactly \( L \) transmitters will be active, and let us denote them by \( T_{a(1)}, \ldots, T_{a(L)} \), where the integer sequence \( s() \) is strictly increasing.

We first note that if \( P_i = 0 \), then \( a_i = a_{i+1} \). Therefore, the set of equations (16), applied for \( i = s(j) \), \( 1 \leq j \leq L \), give:

\[h_{s(j)} \sum_{k=1}^{j} \frac{\mu_{s(k)} - \mu_{s(k-1)}}{a_{s(k)}} = 0, \quad j = 1, \ldots, L, \]

where we have defined \( s(0) \triangleq 0 \). This set of equations allows the recursive evaluation of the \( a_{s(j)} \)’s, starting from \( a_{s(1)} \) and moving upwards. After straightforward calculations, we have that:

\[a_{s(j)} = \frac{1}{\lambda} \mathcal{R}(s(j), s(j-1)), \quad j = 1, \ldots, L, \]

where, to make the exposition more concise, we have defined:

\[\mathcal{R}[i, j] \triangleq \frac{\mu_j - \mu_i}{h_j - h_i}. \]

\[\]
and \( h_0 \triangleq \infty \). Knowing the \( a_s(j) \) allows the calculation of the powers, using (11). By straightforward induction, we have that:

\[
\sum_{k=1}^{j} P_s(k) = \frac{1}{\lambda} \mu_s(h_s(j)) - \frac{\mu_s(j) h_s(j+1)}{h_s(j) - h_s(j+1)}, \quad j = 1, \ldots, L - 1.
\]  
(19)

Also, we have that \( P_{s(L)} = \frac{a_s(L)}{h_s(k)} \sigma^2 \), and using (17) for \( j = L \), we have that

\[
P_{s(L)} = \frac{1}{h_s(L)} \left[ \frac{1}{\lambda} \mathcal{R}[s(L), s(L-1)] - \sigma^2 \right].
\]  
(20)

Substituting (19), for \( j = L - 1 \), and (20) in the sum-power constraint (13), we derive the value for \( \lambda \):

\[
\lambda = \frac{\mu_s(L) h_s(L)}{P_0 h_s(L) + \sigma^2}.
\]  
(21)

The resulting value for the objective is:

\[
(\mu \cdot \mathbf{R})_{\text{max}} = \frac{1}{2} \sum_{j=1}^{L} (\mu_s(j) - \mu_s(j-1)) \log \left( \frac{1}{\lambda} \mathcal{R}[s(j), s(j-1)] \right)
\]

\[
- \frac{1}{2} \mu_s(1) \log(\sigma^2),
\]

which, after substituting for \( \lambda \) and \( \mathcal{R}[s(j), s(j-1)] \) and rearranging terms, becomes:

\[
(\mu \cdot \mathbf{R})_{\text{max}} = \frac{1}{2} \mu_s(L) \log \left( \frac{P_0 h_s(L) + \sigma^2}{\mu_s(L) h_s(L) \sigma^2} \right)
\]

\[
+ \frac{1}{2} \sum_{j=1}^{L} (\mu_s(j) - \mu_s(j-1)) \log \left( \frac{\mu_s(j) - \mu_s(j-1)}{h_s(j)} \right).
\]

F. A set of conditions on the active set

In Section II-E, we calculated the optimal power allocation, assuming the set of active transmitters is known. Here, we derive a set of conditions that uniquely specify this set, and are required so that the KKT conditions (13)-(16) are satisfied.

**Condition A:** \( \frac{P_0}{\sigma^2} > \frac{\mu_s(L) - \mu_s(L-1)}{h_s(L) - h_s(L-1)} \), \( i > s(L) \).

**Proof:** To prove the condition, we substitute \( \lambda \), as determined in (21), in (20), and require that \( P_{s(L)} \) be positive. ■

**Condition B:** \( \frac{P_0}{\sigma^2} \leq \frac{\mu_s(L) - \mu_s(L-1)}{h_s(L) - h_s(L-1)} \), \( i > s(L) \).

**Proof:** We first note that:

\[
\sum_{l=1}^{j} \frac{\mu_l - \mu_{l-1}}{a_l} = \sum_{k=1}^{j} \frac{\mu_s(k) - \mu_s(k-1)}{a_s(k)} = \frac{\lambda}{h_s(j)}.
\]  
(22)

The first equality comes from the fact that \( a_{l+1} = a_l \), if \( P_l = 0 \). The second equality comes from using (17). Using (22) for \( j = L \), together with the fact that \( a_1 = \sigma^2 \) for \( l > s(L) \), (15) for \( i > s(L) \) becomes:

\[
\sigma^2 \geq \frac{1}{\lambda} \mathcal{R}[i, s(L)], \quad i > s(L),
\]

which, after substituting for the value of \( \lambda \), gives the condition. ■

**Condition C:** \( \mathcal{R}[s(j), s(j-1)] \geq \mathcal{R}[i, s(j-1)], \quad s(j-1) < i < s(j), \quad 1 \leq j \leq L. \)

**Proof:** This condition comes from the requirement to satisfy (15) when \( s(j-1) < i < s(j), \; j = 1, \ldots, L \):

\[
0 \geq \frac{P_j}{\sigma^2} \leq \frac{\mu_s(j) - \mu_{s(j-1)}}{h_s(j) - h_s(j-1)} - \lambda.
\]

Next, we note that (24) holds not only for \( s(j+n) \), but also for any \( i \) such that \( s(j+n-1) < i < s(j+n) \):

\[
\mathcal{R}[s(j), s(j-1)] \geq \mathcal{R}[i, s(j-1)], \quad 1 \leq j \leq L - 1, \quad s(j+n-1) < i < s(j+n), \quad 1 \leq n \leq L - j.
\]  
(25)
The proof of (25) follows as the proof of (24), but using \( R[i, s(j + n - 1)] \leq \mathcal{R}[s(j + n), s(j + n - 1)] < \mathcal{R}[s(j + n - 1), s(j + n - 2)] \) instead of \( R[s(j + n), s(j + n - 1)] < \mathcal{R}[s(j + n - 1), s(j + n - 2)] \).

Finally, we note that (24) must hold not only for \( s(j + n) \), but also for any \( i > s(L) \):

\[
\mathcal{R}[s(j), s(j - 1)] > \mathcal{R}[i, s(j - 1)], \quad 1 \leq j \leq L, \quad s(L) < i. \tag{26}
\]

Indeed, combining Conditions A and B we have that:

\[
\frac{\mu_i(s-L)}{\sigma^2} < \frac{\mu_i(s-L)}{\sigma^2} - \frac{\mu_i(s-L)}{\mu_i(s-L)},
\]

and by cross-multiplying and simplifying, we arrive at (26).

Combining (24), (25), and (26), we arrive at Condition E.

\section*{G. The Optimal Power Allocation}

Conditions A to E can be used to uniquely determine the active set. To see this, let us assume that the first \( p \) transmitters of that set, i.e., \( T_{s(1)}, \ldots, T_{s(p)} \) have been somehow determined. If the active set contains more than \( p \) transmitters, then there is a unique transmitter that can be \( T_{s(p+1)} \); it is the transmitter \( i > s(p) \), with the maximum ratio \( \mathcal{R}[i, s(p)] \) — if more than one transmitters have the same ratio, it is the one among them with the largest index (i.e., the one that corresponds to the largest priority). Indeed, no other transmitter can satisfy Conditions C and E simultaneously for \( j = p + 1 \). Note that this argument holds even if \( p = 0 \), i.e., the initial set of transmitters is empty. In this case, \( \mathcal{R}[i, s(p)] = \mathcal{R}[i, 0] = h_i \).

Therefore, the only ambiguity we still have left is regarding whether or not we should add a candidate \( T_{s(p+1)} \) to the set of active transmitters. This ambiguity can be resolved by using Conditions A, B, and D. Indeed, let us assume that:

\[
\frac{\mathcal{T}_0}{\sigma^2} > \frac{\mu_i(p)}{\mu_i(p+1)} - \frac{\mu_i(p)}{\mu_i(p+1)} \cdot \frac{\mu_i(p)}{\mu_i(p+1)}, \tag{27}
\]

Then \( T_{s(p+1)} \) must be added to the set of active transmitters, otherwise Condition B will not hold, for \( i = s(p+1) \). On the other hand, let us assume that (27) does not hold, i.e.,

\[
\frac{\mathcal{T}_0}{\sigma^2} \leq \frac{\mu_i(p)}{\mu_i(p+1)} - \frac{\mu_i(p)}{\mu_i(p+1)} \cdot \frac{\mu_i(p)}{\mu_i(p+1)}.
\]

Then, if we do add \( T_{s(p+1)} \) to the active set, it can not be the last transmitter, because in that case Condition A will not be satisfied. But even if we do add more transmitters, by Condition D it follows that Condition A will not hold no matter when we stop. It follows that \( T_{s(p+1)} \) can not be added to the set of transmitters to conclude, the last transmitter to be added is the last candidate for which (27) holds.

We collect the findings of this section in the following theorem:

\textbf{Theorem 1: Calculation of Candidate Set:} Remove from consideration all transmitters for which there is another transmitter with the same or better channel gain and the same or better priority. If there are transmitters with the same channel gain and priority, keep only one of them arbitrarily. Place the remaining transmitters in terms of increasing priorities (equivalently decreasing channel gains) and denote them by \( T_1, \ldots, T_m \). Initially, let \( K = 0 \) and the candidate set be the empty set.

\textit{Execute the following loop:}

1) Find the unique index \( K’ \) for which:

\[
\mathcal{R}[K’, K] > \mathcal{R}[j, K], \quad K’ < j < m,
\]

and

\[
\mathcal{R}[K’, K] > \mathcal{R}[j, K], \quad K < j < K’.
\]

\textbf{Fig. 2.} \( F(\mathcal{P}_0; \mu, h) \) as a function of \( \mathcal{P}_0 \) for a case where \( m = 10 \).

2) Add \( T_{K’} \) to the candidate set and let \( K = K’ \).

3) If \( K < m \), go to step 1). Otherwise, terminate the loop.

Transmitters not belonging to the candidate set will receive no power. Relabel the transmitters of the candidate set as \( T_1, T_2, \ldots, T_m \), and their priorities and gains as \( \mu_1, \mu_2, \ldots, \mu_m \) and \( h_1, h_2, \ldots, h_m \) respectively.

\textbf{Optimal power allocation:} Set \( L = \max\{i : \frac{\mathcal{P}_0}{\sigma^2} > \frac{\mu_i}{\mu_i - \mu_i - 1} \} \), and let

\[
a_j = \left( \frac{\mathcal{P}_0 h_L + \sigma^2}{\mu_i h_L} \right) ^{1 - \frac{\mu_i}{\mu_i - \mu_i - 1}}, \quad j = 1, \ldots, L,
\]

\[
a_{L+1} = \sigma^2.
\]

The optimal power allocation for the candidate set is given by:

\[
\mathcal{P}_j = \begin{cases} \frac{a_j - a_j}{h_j}, & j = 1, \ldots, L, \\ 0, & j = L + 1, \ldots, m. \end{cases}
\]

\textbf{and the optimum value of the objective is given by:}

\[
F(\mathcal{P}_0; \mu, h) \triangleq (\mu \cdot \mathcal{R})_{\text{max}} = \frac{1}{2} \mathcal{P}_0 \log \left( \frac{\mathcal{P}_0 h_L + 1}{\mu_i h_L} \right) + \frac{1}{2} \sum_{j=1}^L (\mu_j - \mu_{j-1}) \log \left( \frac{\mu_j - \mu_{j-1}}{h_j - h_j} \right). \tag{29}
\]

Observe that, in order to simplify the notation, after we calculate the candidate set we relabel it as \( T_1, T_2, \ldots, T_m \). As discussed, the optimal power allocation is unique, unless one of the transmitters of the candidate set \( T_j \) is receiving positive power and there are other transmitters with the same gain and priority that we have disregarded earlier on. In this case, the power allocation is unique up to an arbitrary transfer of the power \( T_j \) to these transmitters.

The optimal value for the objective, \( F(\mathcal{P}_0; \mu, h) \), is a function of \( \mathcal{P}_0 \) explicitly, through the first term of (29), and also implicitly, through the cutoff index \( L \). Since \( L \) can range between \( 1 \) and \( m - 1 \), to be the power level at which we go from \( L \) to \( L + 1 \) active transmitters. Clearly,

\[
\frac{\mathcal{P}_{L+1}}{\sigma^2} = \frac{\mu_i}{\mu_{i+1} - \mu_{i+1}} h_{L+1}.
\]
Using this equality, it follows by straightforward substitutions that:

\[ F(\mathcal{P}_{L,L+1}; \mu, \mathbf{h}) = F(\mathcal{P}_{L,L+1}^+; \mu, \mathbf{h}), \]
\[ F'(\mathcal{P}_{L,L+1}; \mu, \mathbf{h}) = F'(\mathcal{P}_{L,L+1}^+; \mu, \mathbf{h}), \]

where with \( F' \) we denote the partial derivative of \( F \) with respect to the power. Therefore, the \( m \) logarithmic components are arranged so that \( F(\mathcal{P}_0; \mu, \mathbf{h}) \) is smoothly continuous, and hence strictly concave. An example appears in Fig. 2, where the logarithmic functions are plotted with dotted lines and \( F(\mathcal{P}_0; \mu, \mathbf{h}) \) with a continuous line. Points where the cutoff index \( L \) changes are denoted with a solid circle.

### III. The \( m \)-User Parallel MAC

Let us now assume that the \( m \) transmitters can communicate with the single receiver through any of \( K \) channels. As transmitters using the same channel will be interfering with each other, we have a set of \( K \) parallel MACs. Let \( \mathbf{h}^j \) be the channel gains, \( \mathcal{P}^j = (\mathcal{P}_1^j, \ldots, \mathcal{P}_m^j) \) be the powers allocated to the \( m \) transmitters, and \( \mathcal{P}_0^j = \sum_{i=1}^m \mathcal{P}_i^j \) be the total power, all pertaining to channel \( j \). We assume, with no loss of generality, that the receiver noise power is \( \sigma^2 \) in all channels. (If the noise powers are different, we can rescale the channel gains).

The constraint on the total power for the single channel case now extends to the constraint that the total power over all \( m \) transmitters and \( K \) channels should be less than \( \mathcal{P}_0 \).

The capacity region of this channel was calculated in Theorem 2.1 of [7], and can be expressed as

\[ C_{\text{MAC}}^{\text{S,PL}}(\mathcal{P}_0; \{\mathbf{h}^j\}_{j=1}^K) = \bigcup_{\mathcal{P}^j \leq \mathcal{P}_0} \sum_{j=1}^K \mathcal{C}_{\text{MAC}}^{\text{S}}(\mathcal{P}_0^j; \mathbf{h}^j) \quad (30) \]
\[ = \bigcup_{\mathcal{P}^j \leq \mathcal{P}_0} \sum_{j=1}^K \mathcal{C}_{\text{MAC}}^{\text{S}}(\mathbf{h}^j). \quad (31) \]

where, for two sets \( A \) and \( B, A + B \triangleq \{a + b : a \in A, b \in B\} \). The intuition is straightforward: the rate vectors that are achievable are exactly those that can be written as sums of rate vectors that can be achieved in the individual MACs, under some division of the available power. Therefore, the improvement in the capacity by having multiple channels, comes only from our flexibility in assigning the power to the individual channels.

Note that the capacity region in convex. Indeed, let \( \{\mathcal{P}^j_1\} \) and \( \{\mathcal{P}^j_2\} \) be two distinct distributions of powers among the different transmitters and channels, that achieve the rate vectors \( \mathbf{R}_1 \) and \( \mathbf{R}_2 \) respectively. By a time division argument on the individual MACs, the power allocation \( \alpha \{\mathcal{P}^j_1\} + (1 - \alpha) \{\mathcal{P}^j_2\} \) will achieve the rates \( \alpha \mathbf{R}_1 + (1 - \alpha) \mathbf{R}_2 \), for any \( \alpha \in (0,1) \). It follows by convexity [12], that the boundary of the capacity region can be completely characterized by solving the following problem:

maximize: \( \mu \cdot \mathbf{R} \), subject to: \( \mathbf{R} \in C_{\text{MAC}}^{S,PL}(\mathcal{P}_0; \{\mathbf{h}^j\}_{j=1}^K) \), \quad (32)

for any priority vector \( \mu = (\mu_1, \mu_2, \ldots, \mu_m) \geq 0 \).

To calculate the optimal power allocation, we first observe that Lemma 1 continues to hold, i.e., in all channels, the decoding should be in order of increasing priorities. Indeed, if the decoding in one of the channels is otherwise, we can change it and strictly increase the contribution of this particular channel to the maximum, without changing the contributions of the other channels. With a similar argument, it follows that the distribution of power among the users of a given channel should satisfy Theorem 1 for that channel. Therefore:

\[ (\mu \cdot \mathbf{R})_{\text{max}} = \sum_{j=1}^K F(\mathcal{P}^j_0; \mu, \mathbf{h}^j), \quad (33) \]

for some distribution of powers \( \{\mathcal{P}^j_0\} \) satisfying \( \sum_{j=1}^K \mathcal{P}^j_0 = \mathcal{P}_0 \), and the only thing remaining to find is the distribution \( \{\mathcal{P}^j_0\} \).

To this end, let us concentrate on any two channels \( p \) and \( q \) that are allocated positive powers \( \mathcal{P}^p_0 \) and \( \mathcal{P}^q_0 \). We must have:

\[ F'(p; \mu, \mathbf{h}^p) = F'(q; \mu, \mathbf{h}^q) \leq \lambda > 0. \]

Indeed, if, for example, \( F'(p; \mu, \mathbf{h}^p) < F'(q; \mu, \mathbf{h}^q) \), we can move an incremental amount of power from channel \( p \) to channel \( q \), and thus achieve a further increase in the objective. This contradicts the optimality of the power allocation. For the same reason, it follows that if another channel \( r \) is allocated no power, i.e., \( \mathcal{P}^r_0 = 0 \), then we must have:

\[ F'(0; \mu, \mathbf{h}^q) \leq \lambda. \]

Finally, observe that, by the results of Section II, the derivatives \( F'(\mathcal{P}^j_0; \mu, \mathbf{h}^j) \) are continuous and strictly decreasing, with a finite value at \( \mathcal{P}^j_0 = 0 \), and approaching 0 as \( \mathcal{P}^j_0 \to \infty \). Therefore, it follows that any \( \lambda > 0 \) will uniquely specify a total power and its associated optimal power allocation. In addition, the total power increases as \( \lambda \) decreases. In fact, as \( \lambda \to 0 \), \( \mathcal{P}_0 \to \infty \).

As an example, let us consider the case where there are 4 parallel channels, whose partial derivatives \( F'(\mathcal{P}^j_0; \mu, \mathbf{h}^j) \) are plotted in Fig. 3. If, for example, we fix \( \lambda = 0.1 \), then channel 1 will receive no power, i.e., \( \mathcal{P}^1_0 = 0 \), and channels 2, 3, and 4 will be allocated the powers \( \mathcal{P}^2_0, \mathcal{P}^3_0, \mathcal{P}^4_0 \) at which the partial derivatives of the respective channels become equal to \( \lambda \). The figure clearly shows that, for any other \( \lambda > 0 \) there is a similar, uniquely defined optimal power allocation.

Combining the findings of this section, we have:

Theorem 2: For each \( \mathcal{P}_0 > 0 \) and its associated optimal power allocation that maximizes (32), there is a unique \( \lambda > 0 \), such that the total powers allocated per channel, \( \{\mathcal{P}^j_0\} \), satisfy:

\[ F'(\mathcal{P}^j_0; \mu, \mathbf{h}^j) = \lambda \text{ if } \mathcal{P}^j_0 > 0, \quad (34) \]
\[ F'(0; \mu, \mathbf{h}^j) \leq \lambda \text{ if } \mathcal{P}^j_0 = 0. \quad (35) \]
Within each channel \( j \), the total power \( P_0^j \) is allocated according to Theorem 1. The maximum value of (32) is given by (33).

Note that we are implicitly using a standard Lagrangian formulation. In Lagrangian parlance, \( \lambda \) is the price at which we are buying power. In this context, it makes sense that the total power \( \lambda \) decreases as its price \( \lambda \) increases. As is typically the case in similar problems with a Lagrangian formulation, \( \lambda \) cannot be written as an explicit function of \( P_0 \), at least to the best of our understanding. However, as \( \lambda \) is a monotonically increasing function of \( P_0 \), the value of \( \lambda \) that corresponds to a particular power \( P_0 \) can be found by a simple bisection algorithm [15].

IV. THE \( m \)-USER BROADCAST CHANNEL

A. Single BC Case

The \( m \)-user Gaussian Broadcast Channel (BC) consists of \( m \) receivers \( V_1, V_2, \ldots, V_m \), receiving data from a single transmitter \( T \). At integer time instances \( i \) the transmitter broadcasts a signal \( X[i] \), and the receivers receive the signals:

\[
Y_j[i] = h_j X[i] + n_j[i], \quad j = 1, \ldots, m,
\]

where \( h_j > 0 \) is the constant power gain between \( T \) and \( V_k \), and \( n_j[i] \) is Gaussian noise with zero mean. The noises \( n_j[i] \) are statistically independent, and are assumed to have a common power \( \sigma^2 \). (The more general case where the noises have different powers can be easily brought into this special case, by a rescaling of the received signals.) The transmitter \( T \) can transmit with any power at time index \( i \), as long as its long-term time average is less than \( P_0 \). We use the notation \( h = (h_1, \ldots, h_m) \).

The capacity region \( C_{BC}(P_0; h) \) is the set of all rate vectors \( R = (R_1, R_2, \ldots, R_m) \) such that all receivers \( V_1, V_2, \ldots, V_m \) can simultaneously receive data from the transmitter with rates \( R_1, R_2, \ldots, R_m \), respectively, known to be the following convex set [9]:

\[
\begin{align*}
C_{BC}(P_0; h) &= \left\{ R : \\
R_i &\leq \frac{1}{2} \log_2 \left( 1 + \frac{h_i P_0}{\sigma^2 + h_i \sum_{j=1}^m h_j P_j} \right), \quad i = 1, \ldots, m, \\
\sum_{j=1}^m P_j &= P_0, \quad P_j \geq 0, \quad i = 1, \ldots, m \right\}. \quad (36)
\end{align*}
\]

where \( \pi(\cdot) \) is any permutation of the numbers \( \{1, 2, \ldots, m\} \) that orders the \( m \) receivers in non-decreasing channel quality, i.e., \( h_{\pi(1)} \leq h_{\pi(2)} \leq \cdots \leq h_{\pi(m)} \).

To achieve a point \( R = (R_1, R_2, \ldots, R_m) \) in the capacity region, the transmitter encodes with rate \( R \), the message intended for receiver \( V_i \), independently of the other messages, and transmits it with long-term average power \( P_i \), simultaneously with the signals intended for the other receivers. Each receiver will successively decode the signals for each of the receivers, in the order specified by \( \pi(\cdot) \), i.e., in an order of non-decreasing channel quality stopping after it decodes its own signal. When a receiver decodes a signal, other signals that have already been decoded do not create any interference, but the rest of the signals appear as thermal noise. Although decoding orders that are not according to non-decreasing channel quality are also possible, they will not in general attain points on the boundary of the capacity region.

As with the MAC, it follows by the convexity of the capacity region, that its boundary can be completely traced out by solving the following maximization problem [12]:

\[
\text{maximize: } \mathbf{\mu} \cdot \mathbf{R} \quad \text{subject to: } \mathbf{R} \in C_{BC}(P_0; h), \tag{37}
\]

for all priority vectors \( \mathbf{\mu} = (\mu_1, \mu_2, \ldots, \mu_m) \) such that \( \mu_j \geq 0 \). We could solve this optimization problem starting from first principles, as was done in [1], and as we did in Section II for the MAC. However, the following theorem, introduced recently in [8], gives us a much shorter path:

Theorem 3: Consider the following dual channels:

1) A BC in which the power gains between the transmitter and the \( m \) receivers are \( \mathbf{h} = (h_1, h_2, \ldots, h_m) \), and the receivers are susceptible to thermal noise with a common power \( \sigma^2 \).

2) A MAC in which the power gains between the receiver and the \( m \) transmitters are also \( \mathbf{h} = (h_1, h_2, \ldots, h_m) \), and the receiver is susceptible to thermal noise also of power \( \sigma^2 \).

The capacity region of the BC is equal to the capacity region of the dual MAC under a sum-power constraint:

\[
C_{BC}(P_0; \mathbf{h}) = C_{MAC}^S(P_0; \mathbf{h}). \tag{38}
\]

Furthermore, let \( \mathbf{R} \) be a point in the capacity region of the MAC, achieved if the receiver decodes the incoming signals in the order of increasing index, and the powers available to the transmitters are \( (P_1^{MAC}, P_2^{MAC}, \ldots, P_m^{MAC}) \). The dual BC will achieve the same point in the capacity region if each receiver decodes the incoming signals in the order of decreasing index, stopping after it decodes its own signal, and the powers \( (P_1^{BC}, P_2^{BC}, \ldots, P_m^{BC}) \) allocated to the individual signals are given by:

\[
P_i^{BC} = \frac{P_i^{MAC}}{\sigma^2 + \sum_{j=i+1}^m h_j P_j} \quad i = 1, \ldots, m. \tag{39}
\]

A direct application of Theorem 3 gives:

Theorem 4: To find the maximum of (37), find the candidate set and cutoff point \( L \) as in Theorem 1. Then, set

\[
\beta_j = \frac{\mu_j}{\mu_{j+1}} - \frac{\mu_{j+1}}{\mu_j}, \quad j = 1, \ldots, L, \quad \beta_{L+1} = \frac{\mu_{L+1}}{1}, \quad (40)
\]

Under the optimal power allocation, receivers outside the candidate set receive no power. Receivers in the candidate set receive power according to:

\[
P_i = \left\{ \begin{aligned}
\beta_{j+1} - \beta_j & \quad j = 1, \ldots, L, \\
0 \quad & j = L + 1, \ldots, n,
\end{aligned} \right. \tag{40}
\]

and the maximum is given by (29) of Theorem 1.

**Proof:** The fact that the maximum is given by (29) follows from the equality of the two capacity regions. To prove that the optimal power allocation is given by (40), it suffices to show that (40) and (28) satisfy the transform (39). This can be verified after straightforward substitutions.

As with the case of the MAC, the optimum power allocation is unique, unless for one of the receivers of the candidate set that are allocated positive power, there are other receivers outside the candidate set with the same gain and priority.

B. Parallel BC Case

Let us now assume that the \( m \) receivers can receive information from the transmitter through any of \( K \) independent channels. As transmissions over the same channel will be interfering with each other, we have a set of \( K \) parallel BCs. Let \( \mathbf{h}^k \) be the channel gains, \( P_k = (P_1^k, \ldots, P_m^k) \) be the powers allocated to the \( m \) receivers, and \( P_0 = \sum_{k=1}^m P_k^k \) be the total power, all pertaining to channel \( j \). We assume, with no loss of generality, that the receiver noise powers are \( \sigma^2 \) in all channels and users. (If the noise powers are different, we
can rescale the channel gains). The constraint on the total power for the single channel case now extends to the constraint that the total power over all $m$ receivers and $K$ channels should be at most $P_0$.

The capacity region of this set of parallel BCs was calculated in [16] for the two channel case, and the following straightforward generalization for $K$ channels appears in [3], [8]:

$$C_{BC}^{(K)}(P_0; \{h^k\}_{j=1}^K) = \sum_{j=1}^K C_{BC}^{(1)}(P_0; h^j). \quad (41)$$

The intuition is as in the MAC case: the rate vectors that are achievable are exactly those that can be written as sums of rate vectors that can be achieved in the individual BCs, under some division of the available power. Note that the capacity region of the parallel BCs is identical to the capacity region of the parallel dual MACs. This can be seen by comparing (41) with (30), and using (38).

Similarly to the case of the parallel MACs, the optimal allocation of power within each channel can be found by using Theorem 4. Working as in Section III, the following theorem follows:

Theorem 5: Consider the problem:

$$\text{maximize: } \mu \cdot \mathbf{R} \quad \text{subject to: } \mathbf{R} \in C_{BC}^{(K)}(P_0; \{h^k\}_{j=1}^K). \quad (42)$$

Under the optimal power allocation, within each channel $j$ the total power $P_{0j}$ is allocated according to Theorem 4. Therefore, the maximum value of (42) is given by (33). The total powers allocated per channel, $\{P_{0j}\}$, satisfy (34) and (35) for some $\lambda > 0$.

V. Conclusions

In this correspondence, we calculate optimal power allocations that achieve points at the boundary of the capacity region for sets of parallel Gaussian Multiple Access Channels (MACs) and Broadcast Channels (BCs). In both cases, the optimization is under a constraint on the sum of allocated powers over all users and channels. Our findings are applicable in cases of many-to-one or one-to-many communication over channels that can be modeled as a set of parallel channels, for example channels with fading, Intersymbol Interference (ISI), Time/Frequency/Code Division Multiple Access, etc. However, in order to keep our results as general as possible, we do not place them in the setting of any of these areas. Researchers of these (and other) areas can readily adapt our results to their particular settings.

Throughout this work, we have assumed that the channel gains are known, i.e., we have perfect channel side information (CSI), at all users. The case where there is limited or no CSI has attracted significant interest by the research community, both for single user and multiuser cases (see for example [17], [18] and the references therein), but falls outside the scope of this work.

As a last point, note that we have studied a number of convex capacity regions by finding the maximum values for inner products of the form $\langle \mu \cdot \mathbf{R} \rangle = \sum_{i=1}^m \mu_i R_i$, where the $R_i$ are the rates achieved by individual transmissions, and the coefficients $\mu_i$ are their associated priorities. An alternative interpretation for the $\mu_i$’s is that they are rewards, associated with transmitting a unit of information over a given transmitter-receiver link. Under this interpretation, the sum $\langle \mu \cdot \mathbf{R} \rangle$ represents the total reward of using the channel at the point $\mathbf{R}$. If a reward $\mu_i$ equals the physical distance $x$ between the transmitter and receiver, or, more generally, is a monotonically increasing function of the distance $\mu_i(x)$, the inner product $\langle \mu \cdot \mathbf{R} \rangle$ becomes the transport capacity of the channel. In cases where we are not only interested in transmitting with high rates, but also across large distances, the transport capacity is the natural figure of merit for the efficiency of the communication. This concept has recently attracted significant interest, not only in the context of the BC and MAC [14], [19], but in more general contexts as well [20], [21].

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